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LOGIC THROUGH A LEIBNIZIAN LENS

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1. Introduction

The usual semantics for first-order logic says that the proposition that Fred is blue is true when Fred is in the extension of 'is blue', which I'll simply construe as the set or class of blue things.[1] The semantics involves two commitments:

> **Predicate Extensionality.** The predicate's semantic value is its extension.

> **Predicate Containment.** For true singular propositions, the predicate's semantic value contains the subject's semantic value.

I'll call this standard type of semantics *logical extensionalism* since it treats the truth (or falsity) of a singular proposition as if it depends on whether the predicate's extension contains the subject's semantic value. For ease of expression, I'll call any variant of logical extensionalism an *extensional approach*.

Leibniz favored an inverse approach. His conceptual containment theory (hereafter, "the containment theory") says that the proposition that Fred is blue is true when the concept of Fred contains the concept of being blue.[2] Hence, the containment theory differs from standard extensional approaches in at least two important ways. The first difference concerns the kinds of entities assigned to predicates. To bring this difference into relief, consider coextensional predicates such as 'is a cordate' and 'is a renate' which apply to the same actual individuals.[3] Since the predicates have different meanings but the same extension, extensions alone fail to capture the difference. We can capture the difference, however, if we assign the property or concept of being a cordate to one and the different property or concept of being a renate to the other. Properties and concepts are intensional entities,

1. Though, see Simmons (2000).
2. G VII, 211/P 115. A list of abbreviations for Leibniz's works appears after the main text. For discussions of the containment theory, see Parkinson (1965), Mates (1986), Ishiguro (1990), Adams (1994), Lenzen (2004), and Levey (2011).
3. Quine (1951, 21).

the sorts of entities for which coextensionality doesn't guarantee identity. Since Leibniz assigns concepts rather extensions to predicates, he thereby subscribes to the more general principle below:

> **Predicate Intensionality.** The predicate's semantic value is an intensional entity.

The second difference concerns the direction of the containment relation. To illustrate the difference, consider how you might adjust your understanding of Fred when you learn something new about him. When you learn that Fred is blue, you adjust your understanding of him to include his being blue. Or if you learn that Fred isn't red, you adjust your understanding of Fred to exclude his being red.[4] What anyone would grasp in a perfect understanding of Fred as he actually is would encompass all and only what is truly predicable of Fred. For Leibniz, a name's meaning is a *complete concept*, which is roughly what would be grasped in a perfect understanding of the name's referent (or would-be referent, in the case of a possible individual).[5] A singular proposition is true, then, when the complete concept of the subject contains the concept of the predicate. So Leibniz also subscribes to the converse of Predicate Containment:

> **Subject Containment.** For true singular propositions, the subject's semantic value contains the predicate's semantic value.[6]

Predicate Intensionality and Subject Containment together form an alternative to logical extensionalism. I'll call this alternative *logical intensionalism* and any of its variants an *intensional approach*. It says that a singular proposition is true when the subject's semantic value contains an intensional entity which is the predicate's semantic value.

Logical intensionalism's venerable history includes both Leibniz's containment theory and Richard Montague's natural language semantics.[7] Leibniz and Montague differ most crucially in how they treat intensional entities. Whereas Montague defines intensional entities in relation to their extensions across possible worlds, Leibniz treats intensions as primitive. In this paper, I develop a new semantics for first-order logic inspired by Leibniz's primitive intensionalism. This project is of interest for a few reasons. First, and as I've argued elsewhere,[8] we can use intensions to characterize the shape of possibility space rather than the other way around, as in Montague semantics. Second, if intensions characterize possibility space rather than the other way around, we can use them to capture not only intensional distinctions among coextensive predicates but also hyperintensional distinctions among necessarily coextensive predicates, all within the context of first-order logic. Finally, Leibniz's own primitive intensionalism faces longstanding problems with both relational and quantified propositions. As a result, no intensional approach to first-order logic with primitive intensions has ever been developed. In constructing my semantics, I show that such an approach can overcome these problems.

Now for a preview. In § 2, I further explain the motivation for my new semantics. Then, to clarify the challenges before us, I explain in § 3 how relational and quantified statements pose problems for Leibniz. In the remaining sections, I present the metaphysical backbone of the semantics, offer the semantics itself, and explain how it avoids the problems with relations and quantification that have plagued the containment theory. The resulting semantics solves problems long thought

4. Heim (1983) also captures these adjustments in understanding, but not in a way that respects Predicate Intensionality.

5. In DM 8/AG 41, Leibniz writes: "Thus the subject term must always contain the predicate term, so that one who understands perfectly the notion of the subject would also know that the predicate belongs to it."

6. Leibnizian complete concepts exist in God's mind, but we needn't hold that God exists to endorse Subject Containment. If we distinguish acts of understanding from what is graspable in them, what would be grasped in an act of perfect understanding may exist even if no such act ever does. With Subject Containment, we can also define what it means for an act of understanding 'Fred' to be perfect: it involves grasping all and only the intensional entities contained in the semantic value of 'Fred' as being so contained.

7. Although both theories satisfy Subject Containment, I will continue to refer to Leibniz's theory exclusively as 'the containment theory'.

8. Warmke (2015).

to be unsolvable, can represent hyperintensional distinctions among properties and impossibilities not representable in standard extensional approaches, and yields, at the same time, an account of logical consequence that is extensionally equivalent to accounts of logical consequence in standard extensional approaches to classical first-order logic.

2. Montague and Others

In Montague semantics, a predicate's semantic value is a property, which is defined as a function from possible worlds (and times) to functions from individuals to truth values.[9] Two such functions may assign the same truth values to the same individuals at the same times in the actual world but assign different values to individuals in other possible worlds. So Montagovian properties qualify as intensional entities, and Montague semantics thereby satisfies Predicate Intensionality. Montague semantics also satisfies Subject Containment. A name's meaning is its referent's set of properties, and a singular proposition is true when that set of properties contains the property expressed by the predicate. So 'Fred is blue' is true when the set of Fred's properties contains the property of being blue.

Montague semantics qualifies as an intensional approach, and it would be relatively simple to use a fragment of it for first-order logic. But Montague uses possibility space—the distribution of individuals and their properties across possible worlds—to characterize his intensional entities. Montagovian intensional entities aren't primitively intensional. They are defined set-theoretically over their extensions across possible worlds. But I elsewhere use primitive intensional entities to characterize possibility space. In Warmke (2015), I develop a new, non-Kripkean semantics for propositional modal logic which defines necessity and possibility via inclusion and exclusion relations among primitively intensional entities. Though I don't have the space to explain or defend the modal semantics in detail, I'll highlight the

9. Montague (1970a,b, 1973).

main differences between my approach and standard possible worlds approaches.[10]

Standard possible worlds approaches say that ⌜$\Box\phi$⌝ (read "necessarily, ϕ") is true when ⌜ϕ⌝ is true in all accessible possible worlds. The diagram below crudely depicts (without any accessibility relation) the situation in which a proposition ϕ is true in all possible worlds w_1 through w_n, including the actual world, w_3:

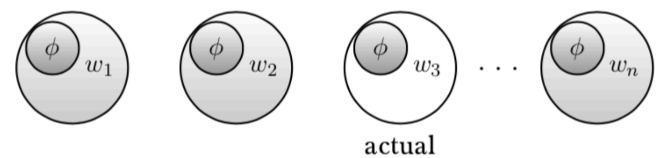

Figure 1. *Necessary truth according to possible worlds approaches.*

My semantics, on the other hand, revolves around two properties. If 'alpha' names the actual world, the first is the property of being alpha, which accounts for the truth of non-modal propositions. So ⌜ϕ⌝ is true when the propositional property of being such that ϕ (hereafter [ϕ]) is part of *being alpha* (hereafter, $\mathcal{A}$). And just as we might say that being a dog in general is part of being Fido, the property of being a world in general is part of being alpha. This more general property of being a world accounts for the truth of modal propositions. So ⌜$\Box\phi$⌝ is true when [ϕ] is part of being a world in general (hereafter, $\mathcal{W}$). The diagram below roughly depicts the situation in which ϕ is necessary:

10. Warmke (2016, 710–712) contains a slightly longer presentation.

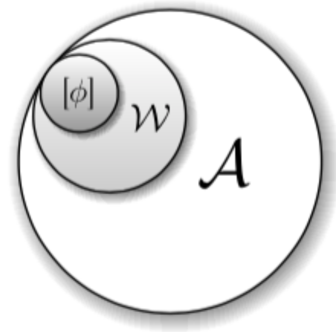

Figure 2. *Necessary truth according to my modal semantics.*

Moreover, instead of using an accessibility relation among worlds to validate or invalidate certain modal principles, I use a property parthood relation. For instance, in possible worlds approaches, the T axiom schema ($\Box\phi \supset \phi$) is valid in models in which every world accesses itself. But in my semantics, the T axiom is valid when every part of $\mathcal{W}$ is part of $\mathcal{A}$. Since $\mathcal{W}$ is part of $\mathcal{A}$ in the models we'd use to capture metaphysical modality, the transitivity of property parthood guarantees that every part of $\mathcal{W}$ is part of $\mathcal{A}$. For if $[\phi]$ is part of $\mathcal{W}$ in any of these models (so that $\ulcorner\Box\phi\urcorner$ is true in them), then $[\phi]$ is also part of $\mathcal{A}$ in those models (so that $\ulcorner\phi\urcorner$ is true in them). The property parthood relation governs primitive intensions and forms the metaphysical backbone of both my modal semantics and my semantics here for first-order logic. I revisit property parthood in § 4.

In the wider metaphysical background, relations among intensional entities explain the shape of modal space and not the other way around. So, for instance, horses are mammals in every possible world simply because *being a mammal* is part of *being a horse*. Since *being a mammal* is part of *being a horse*, nothing could instantiate the property of being a horse without also instantiating the parts of that property, including the property of being a mammal.[11] So all possible horses are mammals. Similarly, p is necessarily true when *being such that p* is part of *being a world (in general)*. If *being such that p* is part of *being a world*, nothing could instantiate the property of being a world without instantiating the parts of that property, including the property of being such that p. So all possible worlds are such that p, as we might say. The mereological structures among intensional entities determine the shape of possibility space. Now, if intensionality determines and is not determined by the shape of possibility space, an intensional approach needn't run on the fuel of possible worlds. An intensional approach can run cleanly without them. I show how this is possible even in the context of extensional first-order logic in §§ 4 and 5.

Whether we should ultimately endorse primitive intensions is a question about which reasonable people can disagree and is beyond the scope of this paper. But if we do endorse them, or if we want to see what theoretical work they can do before we judge their case, the present paper serves as a data point on an evolving scorecard theorists will be unable to assess both fully and fairly for quite a long time. In many ways, possible worlds have been wildly successful. But they also have a half-century head start. A grace period for primitive intensions seems reasonable. And although I've already used primitive intensions in the contexts of modal metaphysics and standard deontic logic,[12] I plan to use them in accounts of sets, numbers, counterfactuals, quantified modal logic, and meaning generally.

Furthermore, I and many others suspect that some necessarily coextensive predicates have different meanings. We are fans of hyperintensionality, in other words, and fans of hyperintensionality may find it valuable that my semantics permits distinctions among necessarily coextensive predicates.[13] The predicates 'is three-sided' and 'is three-

11. And, if we so desire, we can use further relations among primitive intensions to account for why it's impossible for a property to be instantiated without its parts.

12. Warmke (unpublished).

13. It permits hyperintensional distinctions but does not require them. Bealer

angled', for example, share the same extension across possible worlds. So they each receive the same function, the same Montagovian property. But many seem to think these predicates differ in meaning.[14] And if numbers exist necessarily and everything is necessarily self-identical, then the property of being self-identical is necessarily coextensive with the property of being such that the number 5 exists.[15] And if a set and its members exist in all the same possible worlds, then the property of being a human is necessarily coextensive with the property of being both human and the member of a set. The properties in each pair arguably differ, but Montague semantics does not capture these hyperintensional differences because Montagovian properties are identical when their extensions across possible worlds are identical. Montagovian properties are, by definition, non-hyperintensional. My semantics, however, can easily capture hyperintensional distinctions.[16]

Before we examine the containment theory, I want to convey a point about semantic theorizing in general. Depending on our purposes, we may prefer one semantics or another given how well it captures some aspect or conception of meaning. For example, as a Leibnizian, I believe that meaning has at least two components: (i) *intension*, the component related to cognition, reasoning, and the phenomena of analyticity, and (ii) *extension*, the component mediated by an expression's intension and which consists of the object(s), if any, an expression applies to in any possible situation.[17] Given this Leibnizian and also admittedly

(1982, 3), Chierchia and Turner (1988, 263), Menzel (1993, 62–64), and Zalta (1988, 6 ff.) are also motivated to capture hyperintensionality. But their theories and others like them do not use a Leibnizian containment structure within an intensional approach, in my sense.

14. Among the many who think trilaterality and triangularity differ, see Nolan (2014) and Sober (1982).

15. For similar cases, see Marshall (2015, 3) and Plantinga (1976, 146).

16. I return to this point again in § 4.

17. Leibnizian extensions are thus not actual world extensions but extensions across possible worlds. Thus, the "intensions" in Montague and Chalmers (2003, 2012) resemble Leibnizian extensions more closely than they do Leibnizian intensions. Only with Leibnizian intensions does it make sense to say that being a mammal is part of being a dog. For Leibniz, as for me, this is quite literally true. But it isn't true for Montague. Even if we use classes of possible individuals instead of the corresponding functions and construe the subclass relation as a kind of parthood (as in Lewis (1986, 1991) respectively), we get the converse judgement. Since the class of possible dogs forms a subclass of the class of possible mammals, it is the property of being a dog that would be part of the property of being a mammal.

non-standard view of meaning, Montague semantics operates heavily on the side of extension. It assigns not Leibnizian intensions to predicates, but the characteristic functions of Leibnizian extensions. So, as I see it, Montague and I aren't so much competitors as coworkers toiling along different axes of meaning. Or, to borrow another metaphor, the garden has space for more than just my flower alone or Montague's flower alone. We can reject horticultural tyranny and let the flowers bloom. I simply want to show that, contrary to the received consensus, primitive intensionalism is viable and worth further consideration.

3. Containment Theory

Leibniz holds that in true singular affirmatives (e.g., *Socrates is tall*), the concept of the subject contains the concept of the predicate. In singular affirmatives, the concept of the subject is the concept of an individual. And Leibniz famously holds that individual concepts are complete, that "the individual concept of each person contains once and for all everything that will ever happen" to that person.[18] So everything truly predicable of Socrates—that he was snub-nosed, drank hemlock, etc.—is contained in his complete concept.[19] The picture of individual concepts here differs substantially from the more recent one of them as functions splayed out across state-descriptions.[20] Leibniz's individual concepts have "conceptual guts."[21] Each individual concept contains concepts which characterize its individual. And because each complete concept contains everything truly predicable of its individual, complete concepts can account for the truth of any singular affirmative proposition.

18. LA 12.

19. DM 13.

20. Carnap (1947, 181).

21. Cover and O'Leary-Hawthorne (1999, 68).

3.1 Relations

Since relational predicates also truly apply to individuals, complete concepts should account for relational truths, too. But Leibniz never explicitly deals with relational propositions, and the containment theory doesn't straightforwardly extend to them. The problem connects to an objection first expressed by Russell (1937, 12–15) that Leibniz does not successfully "reduce" relational statements to predicate-subject statements.[22] But, as Cover and O'Leary-Hawthorne (1999, 58) claim, Leibniz and his commenters have obscured the issue "by a lack of clarity" about both the target and the kind of reduction at play. Fortunately, we needn't wade into the complex debate about reduction here because the purely formal problem of relations is quite clear. The problem concerns primarily how complete concepts of individuals can account formally for the truth conditions of relational statements in first-order logic.

We can express the problem as a trilemma. Consider a relational statement such as 'A is greater than B'. Whose complete concepts do we use to account for the truth of such a statement? Which serves as the Leibnizian subject: 'A' alone (or 'B' alone), neither, or both? If 'A' alone is the subject, then the statement is true, let's say, when the complete concept of A contains the concept of being greater than B. But this says nothing about B's complete concept. So the treatment counts the relational statement as true when A's complete concept contains the concept of being greater than B even if B's complete concept fails to contain the concept of being something such that A is greater than it. In such a case of inharmonious complete concepts, an aspect of the world has been represented in two incompatible ways.[23] How could we say that 'A is greater than B' is true if B's complete concept, which supposedly captures everything true of B, doesn't capture B's half of the relation? Similar remarks apply if 'B' rather than 'A' serves as the lone subject of the relational statement.

We might say that neither 'A' nor 'B' serves as the subject. But this is a non-starter. If neither A's nor B's complete concept accounts for the relational statement, we have two highly undesirable options. We could say that containment theories are by nature incomplete because complete concepts fail to capture some meaningful relational propositions. Or we could simply admit that containment theories are essentially flawed because they count as meaningless obviously meaningful relational propositions. Neither option helps us update the containment theory for first-order logic.

So maybe we should say that 'A' and 'B' both serve as the subjects of the relational statement. Then, 'A is greater than B' is true when A's complete concept contains the concept of being greater than B, and B's complete concept contains the concept of being something A is greater than. This third strategy precludes inharmonious complete concepts from securing the truth of a relational statement because any relational statement counts as false overall if one or the other complete concept fails to represent its side of the relation. This strategy is obviously the right one, but it nonetheless faces two related obstacles.

Suppose A's complete concept contains *being greater than B*, and we assign B's complete concept to 'B'. Then we are threatened with the result that A's complete concept contains the concept of being greater than B's complete concept, which is nonsense. Quantifying into relational statements exacerbates the problem. Consider the statement that A loves everything. If we draw from a domain of complete concepts for the meanings of logical constants and variables range over the same domain, we are threatened with the result that the statement is true when A's complete concept contains the concept of loving *this* complete concept, *that* complete concept, and all other complete concepts in the domain. We want to use complete concepts to represent that A loves every individual and not that A loves the complete concept of every individual. The formal apparatus in § 5 takes the third strategy but avoids these two obstacles.

22. See Parkinson (1965, 39 ff.), Ishiguro (1990, 101–122), and especially Cover and O'Leary-Hawthorne (1999, Ch. 2).
23. Inharmonious complete concepts appear once more in § 5.5

3.2 *Quantification*

Quantified propositions in general pose problems of an entirely different sort for Leibniz. Whereas we now typically treat a universal affirmative of the form *every A is B* as true when the extension of 'B' contains the extension of 'A', Leibniz says that such a proposition is true when the intension of 'A' contains (in his different sense) the intension of 'B'. Unsurprisingly, Leibniz recognized that these two treatments were inversely related. He writes:

> For when I say *Every man is an animal* I mean that all the men are included amongst the animals; but at the same time I mean that the idea of animal is included in the idea of man. 'Animal' comprises more individuals than 'man' does, but 'man' comprises more ideas or more attributes: one has more instances, the other more degrees of reality; one has the greater extension, the other the greater intension.[24]

In this passage, Wolfgang Lenzen (2004, 11) finds evidence for the following thesis (where 'Int(A)' denotes A's intension, 'Ext(A)' denotes A's extension, and contains$_i$ and contains$_e$ abbreviate "conceptually contains" and "has as a subset" or "extensionally contains," respectively):

Inversion-1. Int(A) contains$_i$ Int(B) iff Ext(B) contains$_e$ Ext(A)

As Lenzen notes, Inversion-1 looks suspicious. Given Leibniz's claim that mutual containment implies identity,[25] it implies the following:

Inversion-2. Int(A) = Int(B) iff Ext(B) = Ext(A)

The right-to-left half of Inversion-2 seemingly conflicts with the idea that different intensional entities may sometimes have the same actual extension. But for Leibniz, Ext(A) and Ext(B) are not sets of actual As and Bs but sets of possible As and Bs. So the right-to-left half of Inversion-2 implies the less problematic but still controversial view that necessarily coextensive intensional entities are identical.[26] We will revisit this issue in § 4.1.

A more pressing issue concerns how Leibniz alleviates the tension between Inversion-1 and his treatment of universal affirmatives. They together seemingly imply that 'Every A is B' is true if and only if every possible A is a possible B. So it appears that universal affirmatives are necessarily true if true at all. But some universal affirmatives are contingently true. How does Leibniz account for the contingently true ones?

For these, Leibniz employs the concept of existing in the actual world, which I'll call "the concept of being actual."[27] Concepts of actual individuals, situations, and actually exemplified properties contain the concept of being actual while concepts of non-actual individuals, situations, and actually unexemplified properties contain the concept of being non-actual. Thus, adding the concept of being actual to the concept of anything non-actual yields conceptual inconsistency. Leibniz exploits this conceptual inconsistency to treat contingently true universal affirmatives.

Consider the contingently true universal affirmative that all renates are cordates. For Leibniz, the concept of being a renate and not a cordate contains the concept of being non-actual.[28] So the concept of being an actual renate and not a cordate is inconsistent. Leibniz can then say that the proposition that all actual renates are cordates is true when the concept of being an actual renate and not a cordate is inconsistent or, equivalently, when the concept of being an actual renate contains the concept of being a cordate.[29] Since this treatment doesn't require

24. NE, IV, xvii, 8, p. 486.
25. C 368/P 58.

26. Whether Leibniz consistently or wholeheartedly endorsed Inversion-2 in a way that would preclude distinct necessarily coextensive properties is less than clear. For example, in NE IV, ii, 1, p. 363, Leibniz distinguishes trilaterality and triangularity as "different aspects of one and the same thing."
27. C 270–273.
28. Adams (1994, 65).
29. Lenzen (2004, 79).

that *being a renate* itself contains *being a cordate*, it doesn't imply that all possible renates are cordates.[30]

Now imagine someone like Obama but slightly heavier. We'll call him "Jack." Is Jack possibly actual?[31] For Leibniz, Jack's non-actuality isn't just tagged externally to the complete concept as if to say, "nothing actual answers to this complete concept." Instead, we find it in the complete concept's very innards. Since something must satisfy a concept's parts to satisfy the concept wholly, Jack cannot be actual and satisfy his own concept without satisfying the concept of being non-actual. Since satisfying the concept of being non-actual would require being non-actual, Jack's actuality would require his non-actuality. But nothing can be both actual and non-actual. So, presumably, Jack is not possibly actual.

Leibniz disagrees, however. Leibniz preserves Jack's possible actuality with his infinite analysis account of contingency. This account implies that something is impossible only if there's a finite proof of its negation.[32] For reasons I lack the space to explain here, Leibniz believes that the proofs for the non-actuality of things like Jack are infinitely long.[33] Therefore, absent any *finite* proof of Jack's being non-actual, Jack counts as possibly actual despite the existence of a proof to the contrary.

The containment theory and the infinite analysis account come as a packaged deal. To preserve the contingency of some universal affirmatives, Leibniz slips the concept of being non-actual into the concepts of non-actualia. Doing so threatens the contingency of what is and isn't actual, but Leibniz rescues that contingency with his infinite analysis account. However, the infinite analysis account doesn't capture an absolute notion of metaphysical possibility. If there is a *proof* for something's negation, it is metaphysically impossible, whether the proof is finite or not. Therefore, anyone who wants to update the containment theory must treat universal affirmatives differently to avoid counting the non-actual as impossible. But how?

To review, the containment theory's use of primitive intensions exposes it to deep problems with both quantified and relational statements. So quantified relational statements are doubly problematic within the containment theory. In § 5, I formulate a new approach to first-order logic with primitive intensions that avoids these problems. But, first, let's cover some background.

4. Property Parthood

Developing a formally adequate intensional approach to first-order logic doesn't require maximal specificity about the metaphysics of intensional entities—no more so than an extensional approach to first-order logic requires a specific metaphysics of material objects.[34] The intensional entities could be Leibnizian concepts or Platonic universals or even fictions. But for brevity's sake, I will call them "properties." With this shorthand, I don't mean to endorse any particular account of properties. Nor do I mean to preclude every single account of their nature which would distinguish them from other entities sometimes called "properties."

I assign properties not only to predicates, but to names, too. Given Subject Containment, then, some properties contain other properties. How should we understand this containment? We might understand property containment as Leibnizian conceptual containment if properties are concepts. Or we can understand containment as the converse of a special property parthood relation if we identify properties with universals.[35] Since property containment is the converse of property

30. But with Inversion-1, it implies that *being an actual renate* and *being an actual cordate* are identical.
31. I agree with Van Inwagen (1980, 424–425), Bennett (2005, 316), and others that possibility and possible actuality amount to the same thing. But Leibniz's views require us to put the argument here in terms of possible actuality.
32. For references and a recent take on Leibniz's theory, see Merlo (2012).
33. AG 19 and C 376/P 66.

34. Stalnaker (1984, 57) argues similarly in defense of a more metaphysically neutral account of possible worlds.
35. For a theory of property parthood best-suited for my purposes, see Warmke (2015, 316–323). The property mereologies in Paul (2002, 2006) are also formally compatible with the intensional approach in the next section, as far as I can tell.

parthood (*being F* is part of *being G* just in case *being G* contains *being F*), an approach is no less intensional for adopting one or the other. In fact, property parthood allows us to describe more concisely the properties which a property contains. We may call these a property's "parts." Although parthood itself is relatively well-understood, property parthood is not. And, as far as I know, property parthood has not been used in a semantics for first-order logic. So before I present the semantics, let's first examine my preferred theory of property parthood.[36]

4.1 The Mereology

I'll use 'proper part' as my primitive mereological notion. The definitions and axioms are transplants from classical extensional mereology applied to properties rather than objects.[37] Lower-case variables (x, y, z) range over properties, not objects.

> **Definition 1**. x is *part* of y iff x is either a proper part of y or identical to y.
> **Definition 2**. x and y *overlap* iff some z is part of both x and y.
> **Definition 3**. x and y are *disjoint* iff x and y do not overlap.
> **Definition 4**. x is a *sum* of the ys iff (each of the ys is part of x and (any z overlaps one of the ys iff z overlaps x)).
> **Asymmetry**. If x is a proper part of y, y isn't a proper part of x.
> **Transitivity**. If x is a proper part of y, and y is a proper part of z, then x is a proper part of z.
> **Weak Supplementation**. If x is a proper part of y, then y has another proper part disjoint from x.
> **Unrestricted Composition**. For any specifiable set of properties whatever, there is a sum of those properties which is itself a property.

Some examples will help illustrate the general picture. *Being a mammal* is part of both *being a dog* and *being a cat*. In fact, *being a mammal* is a proper part of both since it is part of each but identical to neither. By Asymmetry, then, we should expect that neither *being a dog* nor *being a cat* is a proper part of *being an animal*. And, by Transitivity, we should expect that since *being an animal* is a proper part of *being a mammal*, it is also a proper part of *being a dog* and *being a cat*. As a result, these latter two properties overlap. (Non-overlapping or disjoint pairs of properties have no parts in common. Perhaps *being a rock* and *being rational* are disjoint.) *Being a dog* has parts besides *being an animal*. *Being a dog* is a sum of *being an animal* and the rest of its parts, whatever they are.

Weak Supplementation ensures that no proper part of a property overlaps with every other proper part.[38] Since *being an animal* is a proper part of *being a cat*, *being a cat* has at least one proper part disjoint from *being an animal*. *Having a backbone* is one possible candidate. It is a proper part of *being a cat*. But since not all animals have backbones, it is not part of *being an animal*. A wise guy might suggest that a property such as *being composed of organic material* is part of not only *having*

Paul presents her mereology within the context of a bundle theory of ordinary objects. I'm not a bundle theorist, but my intensional approach should appeal to mereological bundle theorists. Paul says that individuals are sums or fusions of properties. So if the properties which compose individuals also characterize them (i.e., if the property of being F is part of an individual just in case the individual is an F), then a bundle theorist can view my approach as a metaphysical semantics in the sense of Sider (2011, 112). Also see fn. 48 for how mereological bundle theory would simplify my semantics.

36. In Warmke (2015, 316–320), I defend my conception of property parthood and distinguish it from others in the literature.

37. The final two axioms in Warmke (2015, 322–323) govern the second-order properties of being certain properties. Since the semantics in § 5 needs no second-order properties, I ignore those axioms here.

38. One might worry that for any properties $P_1, P_2, ..., P_n$, there is a disjunctive property $P_1 \vee P_2 ... \vee ... P_n$ which is part of every one of its disjuncts. If there is universal overlap among properties, then Weak Supplementation fails spectacularly. I suspect that the main motivation for universal overlap comes from a modal analysis of property parthood according to which *being F* is part of *being G* iff necessarily, all Gs are Fs. But I reject the modal analysis partly due to the view I specify in § 2 about the relation between modality and intensionality. For more discussion on this point, see Warmke (2015).

a backbone and *being a cat* but also *being an animal*, which would mean that we haven't pinpointed a genuine case of disjoint proper parts. But if *being composed of organic material* overlaps both *having a backbone* and *being an animal*, subtract it and other similar properties from *having a backbone* to find a remainder disjoint from *being an animal*. That remainder might involve, say, *having a column for an enclosed cord that sends messages from one location to another.* Such a column needn't be composed of organic material. The lesson to draw here is that the way we carve up properties with our linguistic and conceptual resources may make it difficult for us to divvy up a property into its disjoint proper parts.[39]

By Unrestricted Composition, any set of properties forms a sum, which is also a property. Even if some properties are not actually or even possibly co-exemplified, their sum exists all the same. No gold mountains may exist, but the sum of *being gold* and *being a mountain* does. And although round squares cannot exist, the sum of *being round* and *being square* exists. As a result, the present view of property parthood involves a Platonic conception of properties in the sense that allows some properties to exist unexemplified.[40]

And that is not all that the parthood theory requires. The property of being a gold mountain is not a sparse property, in the sense of Lewis (1986: 60), either.[41] It is not in some minimal base of properties which together characterize things completely and non-redundantly. My intensional approach requires a more abundant conception of properties. As I explain in the next subsection, we need non-sparse properties for the values of names. And we will obviously need them for predicates if we interpret any as expressing non-sparse properties.

Now, intensional entities have often been disparaged for not having clear identity conditions.[42] The standard solution, which I set aside in § 2, defines a property in terms of its extension across possible worlds. Instead, I define property identity as complete overlap:

> **Identity**. x is identical to y if and only if every part of x is part of y and every part of y is part of x.

Identity as complete overlap also allows for distinct but necessarily coextensive properties. Different parts, different properties, even if they are necessarily coextensive.[43] Properties such as *being a human* and *being a human and the member of a set* aren't identical if one has a part the other does not. Presumably, the latter but not the former has *being the member of a set* as a part. Or someone could argue as much, anyway, and so distinguish these properties without using impossible worlds.[44]

Although my favored background metaphysics requires Weak Supplementation, Unrestricted Composition, and a Platonic conception of properties, the intensional approach itself does not. But the approach does require an abundant conception of properties, and Unrestricted Composition fits well with that conception. In any case, my semantics needs some property mereology or other, and the formal theory sketched here meets my needs.

In § 2, I set aside Montague semantics primarily because it uses possibility space to define its intensional entities. I set my sights on an intensional approach with primitive intensions instead. Happily, I have not used possibility space to define either the intensional entities here

39. And that's consistent with saying that some of these proper parts are disjunctive. But it is also important to note about disjunctive properties that if *being B or C* is part of *being A*, that *being B* and *being C* aren't thereby parts of *being A*. Such an inference does hold for the Boolean conception of property parthood we find in Zimmerman (1997, 463) and Rosenkrantz and Hoffman (1991, 845).

40. But the intensional approach itself is compatible with a quasi-Aristotelian view according to which all atomic properties are exemplified, even if some non-atomic properties aren't.

41. For more discussion on abundant properties and property parthood see Warmke (2015, 314 ff.).

42. See Quine (1957, 17–19), for example.

43. Zalta (1993, 406–407) avoids Quine's charge with an account of property identity which also permits distinct though necessarily coextensive properties.

44. We can run a similar argument to distinguish some necessarily unexemplified properties. The property of being round and square differs from the property of being red and blue all over as long as they have different parts. Presumably, they do. *Being round* is part of the first and not the second, and *being red* is part of the second but not the first.

or the parthood relation among them. In fact, I have elsewhere used property parthood to define the modal notions.[45] So the mereology here does not sneak in possible worlds through the back door of an approach built on top of it.[46]

4.2 *Complete Properties*

The semantic values of names will be properties which resemble Leibnizian complete concepts in both form and function. An individual's *complete property* captures its individual's monadic and relational features. Relations have posed longstanding problems for Leibniz's approach. And we will need the entire semantic machinery to handle them. But we can at least preview the monadic case: a sentence *a is F* is true just in case *being F* is part of *a*'s complete property.

Since I aim to offer a semantics for classical first-order logic, I will assume that complete properties behave in ways that help secure the theorems of classical logic. Classical logic prohibits anything from being neither F nor not-F. So I will assume that for every predicate F in the chosen language, each individual's complete property either has or lacks the property of being F as a part. Consequently, I will ignore issues about vagueness and various paradoxes. Classical logic also prohibits anything from being both F and not-F. This translates to prohibiting any property from both being and not being part of a complete property. Furthermore, I avoid the negative property of being not-F to account for *a*'s not being F.

Along the way, we will answer some questions about identity. Will the approach secure the Indiscernibility of Identicals? Does its Leibnizian pedigree ultimately commit it to the the Identity of Indiscernibles? These issues are further complicated because identity statements are themselves relational. I'll save these questions about identity for § 5.7, after I've explained how to treat relations generally in § 5.5.

45. Warmke (2015).

46. The point isn't that possible worlds appear nowhere in the background metaphysics. For relations among intensional entities to determine possibility space, possibility space has to exist to be so determined. And the background mereology helps explain this determination: each "possible world" is a property composed of more basic propositional properties. The point here is that the intensional approach in the next section does not ultimately rely on possibility space but on the more fundamental intensional entities which determine that space.

5. **Logical Intensionalism**

Even though the semantics here uses primitive intensions with a kind of Leibnizian containment structure, it avoids the problems we encountered with the containment theory in § 3. We will first specify a language and then skip to the semantics without choosing a particular deductive system. We resolve the formal problem of relations in § 5.5 and the problem with quantification in § 5.6.

5.1 *Language*

I will use a standard language $\mathcal{L}$ whose primitive vocabulary includes a fixed denumerable stock of each of the following:

individual constants : $c_1, c_2, c_3, \ldots$
individual variables: $v_1, v_2, v_3, \ldots$
n-place predicates for each $n > 0$: $A^n_1, A^n_2, A^n_3, \ldots$

To simplify matters, I'll call constants "names" and use lower-case letters from the head of the alphabet (e.g., 'a', 'b', 'c') without subscripts as nicknames in place of certain names. I'll also use lower-case letters from the tail of the alphabet (e.g., 'x', 'y', 'z') as nicknames for certain variables. And I'll use upper-case letters without subscripts or superscripts (e.g., 'F', 'G', and 'R') as nicknames for certain predicates. I will forego function symbols for simplicity's sake.

The vocabulary also includes:

The logical connectives $\neg$ (negation), and $\supset$ (conditional)
The universal quantifier $\forall$
The two-place identity predicate $=$
Punctuation marks (, and)

I will also use metalinguistic variables for expressions in the object language: α (with or without numerical subscripts) for terms, i.e., names and variables; Π for predicates; and ϕ and ψ for well-formed formulas (wffs).[47] Finally, I'll take for granted the usual rules governing the formation of wffs, the definition of a quantifier's scope, and so on.

5.2 *Names*

The interpretation of names isn't a non-stop flight from names to complete properties. A name's interpretation involves a composite function which takes a name to its referent and then gives the referent's complete property. So a model will require a non-empty set of actual individuals (written $\mathcal{D}_i$) and a non-empty set of complete properties (written $\mathcal{D}_s$, since it is the domain for the values of *s*ubjects). And we will need two functions to define the interpretation of names, one from the set A of names in the language to individuals in $\mathcal{D}_i$ and a 1-1 onto function from $\mathcal{D}_i$ to complete properties in $\mathcal{D}_s$:

f_1: A $\rightarrow \mathcal{D}_i$
f_2: $\mathcal{D}_i \rightarrow \mathcal{D}_s$

The function f_1 takes a name and gives an individual i in $\mathcal{D}_i$. Then, f_2 takes the individual i in $\mathcal{D}_i$ and gives i's complete property, [i], in $\mathcal{D}_s$.[48] The interpretation function $\mathcal{I}$ gives the value of a name α via f_1 and f_2:

$$\mathcal{I}(\alpha) = f_2(f_1(\alpha))$$

Through f_1 and f_2, $\mathcal{I}$ assigns to each name a complete property.

47. I will often forego standard use/mention markers when the meaning is clear.
48. A mereological bundle theorist may identify individuals with their complete properties. Doing so would simplify the formalism significantly, since $\mathcal{D}_s$ would collapse into $\mathcal{D}_i$. But I am not a bundle theorist, and most aren't. So we will take the more difficult path precisely because, this time, it is less narrow.

5.3 *Variables*

The value of a variable funnels through a variable assignment:

> g is a variable assignment for a model $\mathcal{M}$ if and only if g is a function that assigns to each variable α in $\mathcal{L}$ an individual i in $\mathcal{D}_i$.

To assign values to variables, we also need a *variable interpretation* $\mathcal{G}$, which takes any variable α and assigns it a value via g and f_2:

$$\mathcal{G}(\alpha) = f_2(g(\alpha))$$

Through g and f_2, $\mathcal{G}$ assigns to each variable a complete property.

We can now define the value of any term and also introduce some simplifying notation. Let $\mathcal{M}$ be a model, $\mathcal{G}$ be a variable interpretation based on a variable assignment g, and α a term. We define $|\alpha|_{\mathcal{M},\mathcal{G}}$, the individual denoted by a term α (relative to $\mathcal{M}$ and $\mathcal{G}$), as follows:

$$|\alpha|_{\mathcal{M},\mathcal{G}} = \begin{cases} f_1(\alpha), & \text{if } \alpha \text{ is a constant} \\ g(\alpha), & \text{if } \alpha \text{ is a variable} \end{cases}$$

And we define $\|\alpha\|_{\mathcal{M},g}$, the value of α (relative to $\mathcal{M}$ and $\mathcal{G}$), as follows:

$$\|\alpha\|_{\mathcal{M},\mathcal{G}} = \begin{cases} \mathcal{I}(\alpha) = f_2(f_1(\alpha)), & \text{if } \alpha \text{ is a constant} \\ \mathcal{G}(\alpha) = f_2(g(\alpha)), & \text{if } \alpha \text{ is a variable} \end{cases}$$

Hence, I'll use single bars as in '$|\alpha|_{\mathcal{M},\mathcal{G}}$' to refer to the individual assigned to α. And I'll use double bars as in '$\|\alpha\|_{\mathcal{M},\mathcal{G}}$' to refer to the complete property assigned to α.

5.4 Predicates

A predicate's instance in an atomic wff $\Pi\alpha_1 \ldots \alpha_n$ provides n different properties. I'll assume that predicates come pre-packaged with numbers assigned to their argument places: 1 to the first argument place, 2 to the second (if there is a second place), and so on, until n is assigned to the last place of an n-ary predicate. Relative to a model $\mathcal{M}$ and a variable interpretation $\mathcal{G}$ based on g, for each atomic wff $\Pi\alpha_1 \ldots \alpha_n$, there is an ordered set $\langle|\alpha_1|_{\mathcal{M},\mathcal{G}}, \ldots, |\alpha_n|_{\mathcal{M},\mathcal{G}}\rangle$ whose m^{th} member is the individual either f_1 or g assigns to the term in the m^{th} argument place of the wff. If the m^{th} term α_m is a name, the m^{th} member in the set is given by f_1; if the m^{th} term is a variable, the m^{th} member in the set is given by the variable assignment g. Let's call the resulting set the *individual set* of a predicate's instance. From the individual set of an n-ary predicate's instance within an atomic wff, we're given a further set of n-many properties. Each of these properties abstracts from a single member of the individual set.

Consider the atomic wff 'Rab' and assume that in our model, f_1(a) = Amy and f_1(b) = Ben. Then, the individual set for the predicate in 'Rab' is ⟨Amy, Ben⟩. Suppose 'Rab' says that Amy loves Ben. The individual set then provides us with two properties: the property of loving Ben (written **love**[_ , Ben]) and the property of being loved by Amy (**love**[Amy, _]). We may assign sets of **love**[...]-properties not only to instances of 'R' but also to instances of other binary predicates. Since we won't have occasion to assign the same kind of property to different predicates in the course of the exposition here, I'll adopt the convention of using property expressions with the predicate letter itself in bold. Hence, we'll simply use '**R**[_ , Ben]' instead of '**love**[_ , Ben]'.

The interpretation function takes an instance of Π within any atomic wff $\Pi\alpha_1 \ldots \alpha_n$ and provides a set of n-many **Π**[...]-properties. Formally, if Π is an n-place predicate in the atomic wff $\Pi\alpha_1 \ldots \alpha_n$, then relative to a variable interpretation $\mathcal{G}$ based on g, $\mathcal{I}(\Pi\alpha_1 \ldots \alpha_n) = \{$**Π**[_ , ..., $|\alpha_n|_{\mathcal{M},\mathcal{G}}$], ..., **Π**[$|\alpha_1|_{\mathcal{M},\mathcal{G}}$, ..., _]$\}$, the set which contains a property abstracted from the individual associated with each argument place, where the individual associated with each argument comes by way of f_1 (if a name occurs in that place) or g (if a variable does).[49]

The interpretation function draws these non-complete properties from a further domain of properties (written $\mathcal{D}_p$, since it is the domain of values for predicates). $\mathcal{D}_p$ contains for each type of property **Π**[...], the **Π**[...]-properties with every permutation (with repetition) of individuals from $\mathcal{D}_i$ relative to each abstracted argument place. So, for example, suppose we want to populate $\mathcal{D}_p$ with 3-ary **H**[...]-properties and the individual domain $\mathcal{D}_i$ has four individuals (a_1, a_2, a_3, and a_4). When we abstract from the first argument place, we form all the permutations for the remaining two "spots." Sixteen such properties result: **H**[_ , a_1, a_1] through **H**[_ , a_1, a_4], **H**[_ , a_2, a_1] through **H**[_ , a_2, a_4], and so on. $\mathcal{D}_p$ then contains forty-eight different **H**[...]-properties, since we have sixteen permutations for each of the three argument places.

Where k is the number of individuals in $\mathcal{D}_i$, the equation below gives the number of n-ary **Π**[...]-properties in $\mathcal{D}_p$ for each **Π**[...]-type:

$$n \cdot k^{n-1}$$

49. It is worth highlighting that unlike the interpretation of predicates in standard extensional treatments, the interpretation function here needs some variable assignment or other to interpret predicate instances. An anonymous referee has helpfully suggested that we think of the interpretation function as a higher-order function that takes a variable interpretation and, with that input, assigns values to constants and predicate instances. No intensional model involves any particular variable interpretation, but the interpretation function in each model is, in a sense, incomplete without supplementing it with some variable interpretation or other. I'm unsure whether this is the best strategy for construing the relationship between the interpretation function and the variable assignment, but I am sure I can't think of any better way right now. At any rate, whatever way we understand the connection will involve some significant departure from standard extensional approaches.

When we factor in arbitrarily large relations with a populous domain of individuals, $\mathcal{D}_p$ will contain a plenitude of properties. This may pose problems for some. But since I've already endorsed an abundant conception of properties, I'm not going to balk now simply because we can calculate their abundance. Additionally, the abundance has some theoretical justification. Suppose no one loves anyone but no one hates anyone either. Then, logical extensionalism assigns the same empty set to both relations and blurs the distinction between loving and hating. But with the extra properties at our disposal, we can say that the properties of loving Ted and hating Ted are distinct, in the domain $\mathcal{D}_p$, and had by none.[50]

We now have all the ingredients to define an intensional model. A model $\mathcal{M} = \langle \mathcal{D}_i, \mathcal{D}_s, \mathcal{D}_p, f_1, f_2, \mathcal{I} \rangle$ is such that:

- $\mathcal{D}_i$ is a non-empty set of actual individuals.
- $\mathcal{D}_s$ is a non-empty set of the complete properties of actual individuals.
- $\mathcal{D}_p$ is a non-empty set of non-complete properties.
- f_1 is a function from the set A of names in $\mathcal{L}$ to $\mathcal{D}_i$
- f_2 is a 1-1 onto function from $\mathcal{D}_i$ to $\mathcal{D}_s$.
- $\mathcal{I}$ obeys the following constraints:

  if α is a name, $\mathcal{I}(\alpha) = f_2(f_1(\alpha))$, a complete property $[i] \in \mathcal{D}_s$.

  If Π is an n-place predicate, then given some g and $\mathcal{G}$, $\mathcal{I}(\Pi\alpha_1 \ldots \alpha_n)$ is the set $\{\mathbf{\Pi}[\_ , \ldots, |\alpha_n|_{\mathcal{M},\mathcal{G}}], \ldots, \mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, \ldots, \_]\} \subseteq \mathcal{D}_p$, and which contains a property abstracted from the individual associated with each argument place in $\Pi\alpha_1 \ldots \alpha_n$, where the individual associated with each argument comes by way of f_1 (if a name occurs in that place) or g (if a variable does).

5.5 Relations

Now we can resolve the formal problem of relations. Each name within an atomic sentence serves as a logical subject, no matter how many argument places the sentence has. Roughly, an atomic sentence is true if and only if each name within that sentence corresponds to some individual's complete property which contains the property abstracted from that individual. An example will illustrate the general idea. Where f_1(a) = Amy, f_1(b) = Ben, f_1(c) = Cal and so $\mathcal{I}$(a) = [Amy], $\mathcal{I}$(b) = [Ben], $\mathcal{I}$(c) = [Cal], Habc is true iff **H**[_ , Ben, Cal] is part of [Amy], **H**[Amy, _ , Cal] is part of [Ben], and **H**[Amy, Ben, _] is part of [Cal]. Notably, if Habc says that Amy is between Ben and Cal, then **H**[_ , Ben, Cal] is the property of being between the individuals Ben and Cal and not the property of being between the complete properties of Ben and Cal. As desired, we avoid the result that the sentence's truth requires that Amy's complete property contains the property of being between the complete properties of Ben and Cal.

My approach systematically handles predicates of any arity. Formally, an atomic wff $\Pi\alpha_1 \ldots \alpha_n$ is true relative to a model $\mathcal{M}$ and a variable interpretation $\mathcal{G}$ (based on an assignment g) just in case for each m, $1 \leq m \leq n$, if $\mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, \ldots, \_ , |\alpha_{m+1}|_{\mathcal{M},\mathcal{G}}, |\alpha_n|_{\mathcal{M},\mathcal{G}}]$ is a member of $\mathcal{I}(\Pi\alpha_1 \ldots \alpha_n)$, then $\mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, \ldots, \_ , |\alpha_{m+1}|_{\mathcal{M},\mathcal{G}}, |\alpha_n|_{\mathcal{M},\mathcal{G}}]$ is part of $\mathcal{I}(\alpha_m)$, the complete property which corresponds to the mth name of the sentence. Each relatum's complete property contains a monadic property with a relational aspect, and the relational aspect concerns the relation from the relatum's point of view.[51]

With the problem of relations now dissolved, we may finally char-

50. I should note that we can rearrange the formalism to appeal to those with a more Aristotelian conception of properties. In extensional models, the interpretation function draws from the domain of individuals to provide the meanings of predicates. The values of predicates are collections of individuals from that domain. The more Aristotelian rearrangement would do something like the inverse of that: the values of predicates would be parts of the complete properties in the domain of complete properties. Directly or indirectly, $\mathcal{D}_s$ would contain all the properties there are. So although the intensional approach here posits many unexemplified properties, logical intensionalism as such does not require them. Therefore, in connection with fn. 48, an Aristotelian bundle theorist could collapse $\mathcal{D}_i$, $\mathcal{D}_s$, and $\mathcal{D}_p$ into one domain and then nix f_1 and f_2.

51. Compare A VI. iv. 1645–46 = C 52/AG 32, L 268. Zalta (2000, §. 8.3) argues that Leibnizian complete concepts contain concepts with relational aspects.

acterize complete properties as advertised in § 4.2. Given the truth conditions for atomic wffs above, an individual's complete property contains all and only what is truly predicable of the individual. Furthermore, since we're providing a semantics for classical first-order logic, we require that every complete property [*i*] in $\mathcal{D}_s$ is such that for any non-complete property **Π**[...] in $\mathcal{D}_p$:

(a) Either **Π**[...] is or is not part of [i], and
(b) It's not the case that **Π**[...] both is and isn't part of [i].

Both conditions are reasonable. And both are necessary in a semantics for classical first-order logic. For, along with the truth conditions for atomic wffs given above, condition (a) helps ensure that there are no truth-value gaps and condition (b) helps ensure that there are no inconsistencies.

Standardly, Amy loves Ben only if the ordered pair ⟨Amy, Ben⟩ is a member of the set of lover-loved pairs. But as soon as we insert that ordered pair into that set, Ben comes along for the ride as being loved by Amy. Classical extensional models are *harmonious*—when such a model says that *a* R-relates to *b*, the same model guarantees that *b* is something such that *a* R-relates to it. Now some may find my truth conditions for atomic wffs objectionable since they license inharmonious models. The present approach has models in which Amy's complete property contains the property of loving Ben without Ben's complete property containing the property of being loved by Amy. In these models, it is false that Amy loves Ben, but the falsity arises from disharmonious complete concepts. Since we're accustomed to the inescapably harmonious models of extensional approaches, we may wonder whether a semantics is deficient if some of its models are inharmonious.

More specifically, one might object that inharmonious models represent impossibilities that a semantics for first-order logic should not be able to represent. But, first, is relational disharmony so clearly impossible? Leibniz thought that (1) relations ultimately reduced to the intrinsic features of monads and that (2) in some sense, God could destroy every other monad though the world would appear to chug along from my dominant monad's point of view.[52] I'm not completely certain that Leibniz was wrong, and, with a slightly different interpretation of the semantics itself, inharmonious models could represent these and similar scenarios. In these inharmonious models, no relational statement would be true. But we could point to what makes each such statement false in order to explain why they seem true to the lonely monad.

Importantly, disharmony always suffices for the falsity of the corresponding relational statement. So even if, as I'm inclined to think, disharmony is metaphysically impossible, inharmonious models simply use these metaphysical impossibilities to capture extra ways for relational statements to be false. We can use standard extensional approaches to represent metaphysical impossibilities, too. And, for some purposes, theorists might prefer a semantics which can represent more rather than fewer kinds of metaphysical impossibilities. Some recent accounts of belief and content, for example, crucially involve impossibilities.[53]

One might also argue that although inharmonious scenarios are metaphysically impossible, they are nonetheless logically possible. On this line of thought, since the property of loving Ben differs from the property of being loved by Amy, we should be able to model formally that Amy has the first while Ben lacks the second. And perhaps it is better that a semantics for first-order logic captures more logically possible scenarios, even if some of them are metaphysically impossible.

In any case, the intensional approach has additional expressive power but still validates all and only the theorems of classical first-order logic, as we shall see. This isn't a trade-off, then, but a win-win.

52. DM 14.
53. See Jago (2014) and the references therein for a number of these accounts.

5.6 Quantification

Everything is such-and-such when the property of being such-and-such is part of each complete property in $\mathcal{D}_s$. But we will need some machinery to flesh out this idea. A variable assignment g_i^x is an *x-variant of g* when it differs at most from g in what individual i it assigns to x. For a variable assignment g and each of its x-variants g_i^x, there is a unique variable interpretation $\mathcal{G}^x_{[i]}$ such that $\mathcal{G}^x_{[i]} = f_2(g_i^x)$. These $\mathcal{G}^x$-variants of $\mathcal{G}$ disagree at most about which complete property $[i]$ is assigned to x but otherwise agree with $\mathcal{G}$ about which variables get which complete properties.

With this machinery, we can say that a wff $\forall x\phi$ is true in a model under a variable interpretation $\mathcal{G}$ (based on g) just in case, for all [i] in $\mathcal{D}_s$, ϕ is true under the variable interpretation $\mathcal{G}^x_{[i]}$. And with this treatment, we dissolve the problem of contingently true universal statements. Consider once more the proposition that all renates are cordates (written '$\forall x(\mathrm{R}x \supset \mathrm{C}x)$'). It is true when every complete property $[i]$ is such that if **R**[_] is part of $[i]$, so is **C**[_]. The sentence's truth does not require that **R**[_] itself is part of **C**[_], as Leibniz's treatment does. So the semantics as such does not imply that all possible Rs are possible Cs. Thus, we do not need to use Leibniz's strategy to avoid the conclusion that all renates are cordates necessarily.

One might object that if any property is part of an individual's complete property, then the individual has that property necessarily or essentially. For, then, if all actual Fs are also Gs, every actual F is a G necessarily or essentially anyway. First, the inference to necessity fails given my account of necessity in Warmke (2015). I claim that a proposition ϕ is necessary when *being such that* ϕ is part of *being a world (in general)*. Presumably, many singular propositions lack this feature. Second, the inference to essentiality fails because I distinguish an individual's *individual property* (whose parts concern what is essential to the individual) from the "larger" complete property of which the individual property is but a proper part.[54] The complete property contains what's true about the individual, beyond what's essential to that individual. So we cannot conclude that Tom is essentially F merely because *being F* is part of his complete property. And so we cannot infer that all Fs are Gs essentially merely from the fact that all Fs are Gs.

5.7 Identity

Our strategy for treating relations provides a way to treat identity, too. Each complete property $[i]$ in $\mathcal{D}_s$ has a unique pair of *identity parts* $=[\,\_\,, i]$ and $=[\,i, \_\,]$. And an identity statement $\alpha = \beta$ is true in a model $\mathcal{M}$ under a variable interpretation $\mathcal{G}$ when $=[\,\_\,, |\beta|_{\mathcal{M},\mathcal{G}}]$ is part of $\|\alpha\|_{\mathcal{M},\mathcal{G}}$ and $=[|\alpha|_{\mathcal{M},\mathcal{G}}, \_\,]$ is part of $\|\beta\|_{\mathcal{M},\mathcal{G}}$. With this treatment, we secure the Indiscernibility of Identicals without also securing the Identity of Indiscernibles.

The Indiscernibility of Identicals says that if individual i_1 and individual i_2 are identical, then whatever is truly predicable of one is truly predicable of the other. Since the parts of an individual's complete property capture everything truly predicable of that individual, we can restate the principle intensionally:

> **Indiscernibility of Identicals**. If i_1 is identical with i_2, every part of $[i_1]$ is part of $[i_2]$ and vice versa.

If i_1 is identical with i_2 and in $\mathcal{D}_i$, then f_2 assigns the same complete property to i_1 and i_2. Since property identity requires complete overlap of parts (§ 4.1), what's truly predicable of i_1 coincides exactly with what's truly predicable of i_2. So in no intensional model do we violate the Indiscernibility of Identicals.

The Identity of Indiscernibles says that, identity aside, if whatever is truly predicable of i_1 is also truly predicable of i_2 and vice versa, i_1 and i_2 are identical. We can also restate this principle intensionally:

> **Identity of Indiscernibles**. If, identity parts aside, every part of $[i_1]$ is part of $[i_2]$ and vice versa, then i_1 is identical with i_2.

54. Arnauld initially seems to have a view like this in his correspondence with Leibniz. See LA 48-49.

Since i_1 is identical with i_2 only if $[i_1]$ is identical with $[i_2]$, the Identity of Indiscernibles holds only if no two complete properties share every part except for their identity parts. But there are intensional models in which different complete properties behave exactly this way. So despite its Leibnizian pedigree, logical intensionalism as such permits non-Leibnizian models which falsify the Identity of Indiscernibles.

Now, because f_2 takes each i in $\mathcal{D}_i$ to the complete property $[i]$ in $\mathcal{D}_s$ which contains $=[\ \_\ , i]$ and $=[\ i, \_\ ]$, we may treat identity in the valuation function more simply. We will say that $\alpha = \beta$ is true relative to a model $\mathcal{M}$ and variable interpretation $\mathcal{G}$ iff $\|\alpha\|_{\mathcal{M},\mathcal{G}} = \|\beta\|_{\mathcal{M},\mathcal{G}}$. We can ask many more questions about the metaphysics of identity within the present context. Although these questions are interesting in their own right, their consideration will only delay us from our goal of showing that a Leibnizian intensional approach is possible. So we will set them aside for now and continue on course.

5.8 Truth

The *valuation function* $V_{\mathcal{M},\mathcal{G}}$ for a model $\mathcal{M}$ and a variable interpretation $\mathcal{G}$ (based on g) is the function that assigns to each wff either 0 or 1, given the constraints below:

(i) For any n-place predicate Π and any terms α_1, ..., α_n, $V_{\mathcal{M},\mathcal{G}}(\Pi\alpha_1 \ldots \alpha_n) = 1$ iff for each m, $1 \leq m \leq n$, if $\mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, \ldots, \_\ , |\alpha_{m+1}|_{\mathcal{M},\mathcal{G}}, |\alpha_n|_{\mathcal{M},\mathcal{G}}]$ is a member of $\mathcal{I}(\Pi\alpha_1 \ldots \alpha_n)$, then it is part of $\|\alpha_m\|_{\mathcal{M},\mathcal{G}}$.

(ii) $V_{\mathcal{M},\mathcal{G}}(\alpha = \beta) = 1$ iff $\|\alpha\|_{\mathcal{M},\mathcal{G}} = \|\beta\|_{\mathcal{M},\mathcal{G}}$.

(iii) For any wffs, ϕ, ψ, and any variable α:

(iiia) $V_{\mathcal{M},\mathcal{G}}(\neg\phi) = 1$ iff $V_{\mathcal{M},\mathcal{G}}(\phi) = 0$

(iiib) $V_{\mathcal{M},\mathcal{G}}(\phi \supset \psi) = 1$ iff either $V_{\mathcal{M},\mathcal{G}}(\phi) = 0$ or $V_{\mathcal{M},\mathcal{G}}(\psi) = 1$

(iiic) $V_{\mathcal{M},\mathcal{G}}(\forall\alpha\phi) = 1$ iff for all [i] in $\mathcal{D}_s$, $V_{\mathcal{M},\mathcal{G}^{\alpha}_{[i]}}(\phi) = 1$.

A wff ϕ is an *intensional logical consequence* of the wffs in set Γ ("$\Gamma \vDash_i \phi$") when, for any intensional model $\mathcal{M}_i$ and any variable interpretation $\mathcal{G}$ on $\mathcal{M}_i$, if every wff ψ in Γ is such that $V_{\mathcal{M}_i,\mathcal{G}}(\psi) = 1$, then $V_{\mathcal{M}_i,\mathcal{G}}(\phi) = 1$.[55] A wff ϕ is *intensionally valid* ("$\vDash_i \phi$") when, for any intensional model $\mathcal{M}_i$ and any variable interpretation $\mathcal{G}$ on $\mathcal{M}_i$, $V_{\mathcal{M}_i,\mathcal{G}}(\phi) = 1$. And a wff ϕ is *intensionally satisfiable* when $V_{\mathcal{M}_i,\mathcal{G}}(\phi) = 1$ for some model $\mathcal{M}_i$ and variable interpretation $\mathcal{G}$.

In the Appendix, I prove that my account of logical consequence is extensionally equivalent to the account of logical consequence in a standard extensional approach for classical first-order logic. It follows that my semantics captures the strong soundness and completeness of a standard deductive system for classical first-order logic.

Technically, the approach as I've presented it does not qualify as an intensional approach. In § 1, I define an intensional approach as one which satisfies both Predicate Intensionality (a commitment to assigning an intensional entity to the predicate of a singular proposition or sentence) and Subject Containment (a commitment to treating singular propositions or sentences as true when the semantic value of the subject contains the semantic value of the predicate). However, my approach assigns sets of properties to each atomic sentence or, as I prefer to think, to the instance of the predicate contained within. My approach, for example, assigns the singleton set $\{\mathbf{F}[\_]\}$ to the instance of the predicate 'F' within the sentence 'Fa'. Sets aren't intensional entities, so my approach violates Predicate Intensionality. What's more, the sets which are the values of instances of predicates are not parts of (or contained in) complete properties. Hence, my approach violates Subject Containment, too.

However, we find a similar situation in standard extensional approaches. These standardly assign a set of n-tuples to each n-ary predicate and hence assign a set of 1-tuples to each 1-ary predicate. So the semantic value of each monadic predicate is not the predicate's extension, e.g., {Bill, John, Suzy}, but the set of unit sets of those individuals, e.g., {⟨Bill⟩, ⟨John⟩, ⟨Suzy⟩}—a violation of Predicate Exten-

55. We begin to use 'i' as a subscript here to distinguish these notions from their extensional analogues in the next section.

sionality.[56] Thus, if a name's semantic value is an individual and not the individual's singleton set, the predicate's semantic value doesn't contain the subject's semantic value in any true singular proposition or sentence—a violation of Predicate Containment.

Each pair of violations results from adopting a general principle for predicates of any arity. To formulate such a general principle, one adopts the useful convention of treating monadic predicates as if they were 1-ary relational predicates. Some textbook presentations of extensional approaches forego that convention and treat monadic and relational predicates differently by assigning sets of individuals instead of sets of 1-tuples to monadic predicates. I could similarly treat monadic predicates as a special case and assign a monadic property instead of its singleton to each instance of a monadic predicate. The resulting approach would satisfy Predicate Intensionality since the meaning of the predicate in a singular proposition or sentence would be a property. And since, in a true singular proposition or sentence, the subject's value would contain the property which is the predicate's value, the approach would thereby satisfy Subject Containment, too. My semantics and the one just sketched here do not differ substantially. In both my approach and the one just sketched, 'Fa' is true iff the value of 'a' contains the property **F**[_]. The sets in mine merely serve as scaffolding to prop up properties and ensure that relations among them determine the proper truth conditions.

6. Conclusion

Much more could be said about how my semantics differs from standard extensional approaches. And even more could be said by way of objection and defense. But our goal has been to show that, however implausible some may find its underlying metaphysics, an intensional approach to first-order logic without worlds is *possible*. Like Leibniz's containment theory, my semantics uses primitive intensions. Yet unlike the containment theory, my semantics covers relational sentences, quantified sentences, and quantified relational sentences. It handles contingently true universal affirmatives without a Leibnizian account of contingency and can represent both hyperintensional distinctions among properties and a certain kind of apparently impossible situation that extensional approaches cannot. The approach also captures the strong soundness and completeness of first-order logic.

What lessons, if any, might we draw from this exercise? For decades, philosophers have treated intensionality set-theoretically with possible worlds and their inhabitants. Intensionality has been extensionalized in one way or another largely because Kripke and others built their semantic approaches to modal logic on the foundation of logical extensionalism. Since intensionality wasn't already built in on the ground floor in interpretations of first-order logic, many saw the worlds of possible worlds semantics, on the next floor up, as a useful tool to illuminate intensional phenomena.[57] And this tradition continues for the most part even among fans of hyperintensionality, in one way or another.

Perhaps we've been working upside down. For, as Menzel (1993, 62) says, "one would think that the foundations of intensionality shouldn't require such a heavy-handed modal metaphysics." My semantics for first-order logic builds intensionality into the ground floor where intensional entities can roam freely without being bolted down by actual or even possible extensions. Intensional entities instead bolt down possible extensions. If we have primitive intensions, we needn't look to modal logic or the possible worlds typically used in its treatment to capture intensionality or even hyperintensionality. In fact, modal logic itself can run on primitive intensions instead of possible worlds, as I've argued elsewhere. These points suggest that philosophy might have taken a different path, one in which primitive intensions and not

56. Logicians may adopt the convention that a unit set is identical to its lone member, in which case a set of unit sets and the set of the members of those unit sets are identical. But the convention implies that each unit set is a member of itself and thus conflicts with the Axiom of Regularity in Zermelo-Fraenkel set theory. See Jech (2003, 63).

57. See Perry (1998) and Nolan (2014).

possible worlds played the leading role in theories of meaning and modality. So we may want to reconsider the practice of using possible worlds to characterize intensionality. Doing so would open up new avenues for discussions which have been framed largely in the language of possible worlds.[58]

Appendix A. Equivalence Proof

Here, I prove that my theory's account of logical consequence is extensionally equivalent to the account of logical consequence in a standard extensional approach. Where "$\Gamma \vDash_e \phi$" and "$\Gamma \vDash_i \phi$" say that ϕ is an extensional consequence and intensional consequence of a set of formulas Γ, we will prove:

Equivalence. $\Gamma \vDash_e \phi$ iff $\Gamma \vDash_i \phi$.

To prove Equivalence, we first prove two theses about the relation between intensional and extensional models. Where "$\mathcal{M}_e$, $g \vDash_e \phi$" says that extensional model $\mathcal{M}_e$ satisfies ϕ under assignment g and "$\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \phi$" says that intensional model $\mathcal{M}_i$ satisfies ϕ under a variable interpretation $\mathcal{G}$ based on assignment g:

Models-EI. For each extensional model $\mathcal{M}_e$, there is an intensional model $\mathcal{M}_i$ such that for some assignment g, $\mathcal{M}_e$, $g \vDash_e \phi$ iff $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \phi$.

Models-IE. For each intensional model $\mathcal{M}_i$, there is an extensional model $\mathcal{M}_e$ such that for some assignment g, $\mathcal{M}_e$, $g \vDash_e \phi$ iff $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \phi$.

58. I'm grateful for the many who have offered comments on previous drafts, especially Robert Adams, Keith Simmons, Thomas Hofweber, Laurie Paul, Bill Lycan, David Buller, Dane Hewitt, Lenny Clapp, Geoff Pynn (who suggested a Straussian reading of the paper), and a string of patient referees. There are many others I've forgotten, I'm sure. Please forgive me for my lapse in memory here, as well as the mistakes which surely remain. Finally, I owe a special debt to Laura Warmke and Brandon Warmke for encouragement and moral support.

In other words, each extensional model agrees with some intensional model about what's satisfiable, and each intensional model agrees with some extensional model about what's satisfiable.

Before we prove Models-EI and Models-IE, we first need a standard extensional approach. Let an extensional model M_e be an ordered pair $\langle \mathcal{D}, \mathcal{I}_e \rangle$ such that $\mathcal{D}$ is a non-empty set of individuals and the interpretation function $\mathcal{I}_e$ is such that:

if α is a constant, then $\mathcal{I}_e(\alpha) \in \mathcal{D}$
if Π is an n-place predicate, then $\mathcal{I}_e(\Pi)$ is an n-place relation defined over $\mathcal{D}$.

The denotation of a term α borrows our definition of an assignment g.

$$|\alpha|_{\mathcal{M}_e,g} = \begin{cases} \mathcal{I}_e(\alpha), & \text{if } \alpha \text{ is a constant} \\ g(\alpha), & \text{if } \alpha \text{ is a variable} \end{cases}$$

The *valuation function* $V_{\mathcal{M}_e,g}$ for a model $\mathcal{M}_e$ and a variable assignment g is the function that assigns to each wff either 0 or 1, given the constraints below:

(i*) For any n-place predicate Π and any terms α_1, ..., α_n, $V_{\mathcal{M}_e,g}(\Pi\alpha_1 \ldots \alpha_n) = 1$ iff $\langle\, |\alpha_1|_{\mathcal{M}_e,g}, \ldots, |\alpha_n|_{\mathcal{M}_e,g} \rangle \in \mathcal{I}_e(\Pi)$
(ii*) $V_{\mathcal{M}_e,g}(\alpha = \beta) = 1$ iff $|\alpha|_{\mathcal{M}_e,g} = |\beta|_{\mathcal{M}_e,g}$.
(iii*) For any wffs, ϕ, ψ, and any variable α:
  (iiia*) $V_{\mathcal{M}_e,g}(\neg\phi) = 1$ iff $V_{\mathcal{M}_e,g}(\phi) = 0$
  (iiib*) $V_{\mathcal{M}_e,g}(\phi \supset \psi) = 1$ iff either $V_{\mathcal{M}_e,g}(\phi) = 0$ or $V_{\mathcal{M}_e,g}(\psi) = 1$
  (iiic*) $V_{\mathcal{M}_e,g}(\forall\alpha\phi) = 1$ iff for every $u \in \mathcal{D}$, $V_{\mathcal{M}_e,g^{\alpha}_{u}}(\phi) = 1$

A wff ϕ is an *extensional logical consequence* of the wffs in set Γ ("$\Gamma \vDash_e \phi$") when, for any extensional model $\mathcal{M}_e$ and any assignment g, if every wff ψ in Γ is such that $V_{\mathcal{M}_e,g}(\psi) = 1$, then $V_{\mathcal{M}_e,g}(\phi) = 1$. A wff ϕ is *extensionally valid* ("$\vDash_e \phi$") when, for any extensional model $\mathcal{M}_e$ and any assignment g, $V_{\mathcal{M}_e,g}(\phi) = 1$. And a wff ϕ is *extensionally satisfiable* when $V_{\mathcal{M}_e,g}(\phi) = 1$ for some extensional model $\mathcal{M}_e$ and assignment g.

With the extensional approach now in our pocket, we can begin to prove Models-EI.

A.1 Proof of Models-EI

Each extensional model $\mathcal{M}_e$ includes a domain $\mathcal{D}$ and an interpretation function $\mathcal{I}_e$. The domain $\mathcal{D}$ of individuals in the extensional model $\mathcal{M}_e$ is identical to the domain of individuals $\mathcal{D}_i$ in some class of intensional models. We will call these intensional models *base equivalent*.

The function f_2 in the intensional approach is a bijection from $\mathcal{D}_i$ to complete properties in $\mathcal{D}_s$. So for any extensional model $\mathcal{M}_e$, there is a bijection from its domain $\mathcal{D}$ to the domain of complete properties $\mathcal{D}_s$ in base equivalent intensional models.

Now, the interpretation function $\mathcal{I}_e$ in any extensional model $\mathcal{M}_e$ assigns individuals in $\mathcal{D}$ to constants in $\mathcal{L}$. For each extensional model $\mathcal{M}_e$, where α is any constant, there is a subclass of base equivalent intensional models according to which $\mathcal{I}_e(\alpha) = i \in \mathcal{D}$ iff $f_1(\alpha) = i \in \mathcal{D}_i$. For these *name equivalent* intensional models, $\mathcal{I}_e(\alpha) = i \in \mathcal{D}$ iff $\mathcal{I}_i(\alpha) = [i] \in \mathcal{D}_s$.[59] Of these name equivalent intensional models, at least one is also *atomic equivalent*, or such that for any n-place predicate Π and any terms $\alpha_1, ..., \alpha_n$, $\langle |\alpha_1|_{\mathcal{M}_e,g}, ..., |\alpha_n|_{\mathcal{M}_e,g}\rangle \in \mathcal{I}_e(\Pi)$ iff for each m, $1 \leq m \leq n$, $\mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, ..., \_ , |\alpha_{m+1}|_{\mathcal{M},\mathcal{G}}, |\alpha_n|_{\mathcal{M},\mathcal{G}}] \in \mathcal{I}_i(\Pi\alpha_1 ... \alpha_n)$ is part of $\|\alpha_m\|_{\mathcal{M},\mathcal{G}}$.[60]

59. From now on, I use 'i' as a subscript to distinguish the intensional interpretation function from its extensional counterpart, $\mathcal{I}_e$.

60. As long as some n-ary atomic wff is false in $\mathcal{M}_e$ and $n > 1$, $\mathcal{M}_e$ will have more than one atomic equivalent intensional model since the existence of inharmonious models provides more ways for atomic formulas to be false. But here is a function which defines a single atomic equivalent intensional model from any extensional model. Given $\mathcal{M}_e$, let $\mathcal{M}_i$ be such that $\mathcal{D}_i = \mathcal{D}$, and such that for any n-place predicate Π and any terms $\alpha_1, ..., \alpha_n$, (i) $\langle |\alpha_1|_{\mathcal{M}_e,g} ... |\alpha_n|_{\mathcal{M}_e,g}\rangle \in \mathcal{I}_e(\Pi)$ iff for each m, $1 \leq m \leq n$, $\mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, ..., \_ , |\alpha_{m+1}|_{\mathcal{M},\mathcal{G}}, |\alpha_n|_{\mathcal{M},\mathcal{G}}] \in \mathcal{I}_i(\Pi\alpha_1 ... \alpha_n)$ is part of $\|\alpha_m\|_{\mathcal{M},\mathcal{G}}$, and (ii) $\mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, ..., \_ , |\alpha_{m+1}|_{\mathcal{M},\mathcal{G}}, |\alpha_n|_{\mathcal{M},\mathcal{G}}]$ is part of $\mathcal{I}_i(\alpha_m)$ for some m such that $1 \leq m \leq n$, iff for every m, $1 \leq m \leq n$, $\mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, ..., \_ , |\alpha_{m+1}|_{\mathcal{M},\mathcal{G}}, |\alpha_n|_{\mathcal{M},\mathcal{G}}]$ is part of $\mathcal{I}_i(\alpha_m)$. This gives us the harmonious atomic equivalent intensional model.

In summary, every extensional model has an at least one intensional model which is atomic equivalent and therefore both base and name equivalent. And now we may prove by induction that each extensional model $\mathcal{M}_e$ has some atomic equivalent intensional model $\mathcal{M}_i$ such that for some assignment g, $\mathcal{M}_e$, $g \vDash_e$ A iff $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i$ A. Let's start with the base case of atomic wffs:

When A is $\Pi\alpha_1...\alpha_n$

Since $\mathcal{M}_e$ and $\mathcal{M}_i$ are atomic equivalent, $V_{\mathcal{M}_e,g}(\Pi\alpha_1 ... \alpha_n) = 1$ iff $V_{\mathcal{M}_i,\mathcal{G}_g}(\Pi\alpha_1 ... \alpha_n) = 1$, by (i) and (i*). Therefore, for every atomic wff $\Pi\alpha_1...\alpha_n$, $\mathcal{M}_e$, $g \vDash_e \Pi\alpha_1...\alpha_n$ iff $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \Pi\alpha_1...\alpha_n$.

When A is $\alpha = \beta$

Since $\mathcal{M}_e$ and $\mathcal{M}_i$ are atomic and hence name equivalent, $\mathcal{I}_e(\alpha) = \mathcal{I}_e(\beta) \in \mathcal{D}$ iff $\mathcal{I}_i(\alpha) = \mathcal{I}_i(\beta) \in \mathcal{D}_s$. Therefore, $V_{\mathcal{M}_e,g}(\alpha = \beta) = 1$ iff $V_{\mathcal{M}_i,\mathcal{G}_g}(\alpha = \beta) = 1$, by (ii) and (ii*). So for every wff $\alpha = \beta$, $\mathcal{M}_e$, $g \vDash_e \alpha = \beta$ iff $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \alpha = \beta$.

For the inductive hypothesis (IH), assume that for every formula B less complex than formula A below, $\mathcal{M}_e$ is such that for some atomic equivalent intensional model $\mathcal{M}_i$ and some assignment g, $\mathcal{M}_e$, $g \vDash_e$ B iff $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i$ B. And for the inductive step, we show that the same holds for any (non-atomic) wff A, assuming IH. We proceed by cases of A's main operator. (For $\neg\phi$ and $\phi \supset \psi$, I will prove the left-to-right direction only, since the right-to-left direction is symmetrical to it.)

When A is $\neg\phi$

Suppose $\mathcal{M}_e$, $g \vDash_e \neg\phi$. Then, $V_{\mathcal{M}_e,g}(\neg\phi) = 1$. So $V_{\mathcal{M}_e,g}(\phi) = 0$, by (iiia*). By (IH), $V_{\mathcal{M}_i,\mathcal{G}_g}(\phi) = 0$. Therefore, $V_{\mathcal{M}_i,\mathcal{G}_g}(\neg\phi) = 1$, by (iiia). Hence, $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \neg\phi$.

When A is $\phi \supset \psi$
Suppose $\mathcal{M}_e$, $g \vDash_e \phi \supset \psi$. Then, $V_{\mathcal{M}_e,g}(\phi \supset \psi) = 1$. So either $V_{\mathcal{M}_e,g}(\phi) = 0$ or $V_{\mathcal{M}_e,g}(\psi) = 1$, by (iiib*). By (IH), either $V_{\mathcal{M}_i,\mathcal{G}_g}(\phi) = 0$ or $V_{\mathcal{M}_i,\mathcal{G}_g}(\psi) = 1$. Then, by (iiib), $V_{\mathcal{M}_i,\mathcal{G}_g}(\phi \supset \psi) = 1$, and $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \phi \supset \psi$.

When A is $\forall\alpha\phi$
Suppose $\mathcal{M}_e$, $g \vDash_e \forall\alpha\phi$. Then, $V_{\mathcal{M}_e,g}(\forall\alpha\phi) = 1$ and for every $u \in \mathcal{D}$, $V_{\mathcal{M}_e,g^\alpha_u}(\phi) = 1$, by (iiic*). Given the variable assignment g and each of its α-variants g^α_i, there is a variable interpretation $\mathcal{G}$ whose $\mathcal{G}^\alpha$-variants are such that $\mathcal{G}^\alpha_{[i]} = f_2(g^\alpha_i)$. By (IH), then, every $u \in \mathcal{D}$ is such that $V_{\mathcal{M}_e,g^\alpha_u}(\phi) = 1$ iff every [u] in $\mathcal{D}_s$ is such that $V_{\mathcal{M}_i,\mathcal{G}^\alpha_{[u]}}(\phi) = 1$. Therefore, $V_{\mathcal{M}_i,\mathcal{G}}(\forall\alpha\phi) = 1$ and $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \forall\alpha\phi$.

Suppose $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \forall\alpha\phi$. Then, $V_{\mathcal{M}_i,\mathcal{G}}(\forall\alpha\phi) = 1$, and every [u] in $\mathcal{D}_s$ is such that $V_{\mathcal{M}_i,\mathcal{G}^\alpha_{[u]}} = 1$, by (iiic). The variable interpretation $\mathcal{G}$ is based on the assignment g so that any $\mathcal{G}^\alpha$-variants $\mathcal{G}^\alpha_{[i]}$ and any α-variants g^α_i are such that $\mathcal{G}^\alpha_{[i]} = f_2(g^\alpha_i)$. Then, by IH, every [u] in $\mathcal{D}_s$ is such that $V_{\mathcal{M}_i,\mathcal{G}^\alpha_{[u]}}(\phi) = 1$ iff every $u \in \mathcal{D}$ is such that $V_{\mathcal{M}_e,g^\alpha_u}(\phi) = 1$. So for every $u \in \mathcal{D}$, $V_{\mathcal{M}_e,g^\alpha_u}(\phi) = 1$, and, by (iiic*), ${}_{\mathcal{M}_e,g}(\forall\alpha\phi) = 1$. So $\mathcal{M}_e$, $g \vDash_e \forall\alpha\phi$.

By induction, then, each extensional model $\mathcal{M}_e$ has some atomic equivalent intensional model $\mathcal{M}_i$ such that for some assignment g, $\mathcal{M}_e$, $g \vDash_e \phi$ iff $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \phi$. This establishes MODELS-EI.

A.2 Proof of Models-IE

Each intensional model $\mathcal{M}_i$ includes a domain $\mathcal{D}_i$ of individuals and is identical to the domain of individuals $\mathcal{D}$ in some class of base equivalent extensional models. The function f_2 in the intensional approach is a bijection from $\mathcal{D}_i$ to complete properties in $\mathcal{D}_s$. The inverse of a bijective function is itself bijective, so there is also a bijective function from $\mathcal{D}_s$ in $\mathcal{M}_i$ to $\mathcal{D}_i$. So for any intensional model $\mathcal{M}_i$, there is a bijection from its domain of complete properties $\mathcal{D}_s$ to the domain $\mathcal{D}$ of base equivalent extensional models.

Now, the interpretation function $\mathcal{I}_i$ in any intensional model $\mathcal{M}_i$

assigns complete properties in $\mathcal{D}_s$ to constants in $\mathcal{L}$. For each intensional model $\mathcal{M}_i$, where α is any constant, there is a subclass of base equivalent extensional models according to which $f_1(\alpha) = i \in \mathcal{D}_i$ iff $\mathcal{I}_e(\alpha) = i \in \mathcal{D}$. These name equivalent extensional models are such that $\mathcal{I}_i(\alpha) = [i] \in \mathcal{D}_s$ iff $\mathcal{I}_e(\alpha) = i \in \mathcal{D}$. And of these name equivalent extensional models, one is atomic equivalent, or such that for any n-place predicate Π and any terms $\alpha_1, ..., \alpha_n$, $\langle |\alpha_1|_{\mathcal{M}_e,g}, ..., |\alpha_n|_{\mathcal{M}_e,g}\rangle \in \mathcal{I}_e(\Pi)$ iff for each m, $1 \leq m \leq n$, $\mathbf{\Pi}[|\alpha_1|_{\mathcal{M},\mathcal{G}}, ..., \_\, , |\alpha_{m+1}|_{\mathcal{M},\mathcal{G}}, |\alpha_n|_{\mathcal{M},\mathcal{G}}] \in \mathcal{I}_i(\Pi\alpha_1 ... \alpha_n)$ is part of $\|\alpha_m\|_{\mathcal{M},\mathcal{G}}$.[61]

In summary, every intensional model has at least one extensional model which is atomic equivalent and therefore both base and name equivalent. We may now prove by induction that for each intensional model $\mathcal{M}_i$, some atomic equivalent extensional model $\mathcal{M}_e$ is such that for some assignment g, $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i$ A iff $\mathcal{M}_e$, $g \vDash_e$ A. We'll begin with atomic wffs.

When A is $\Pi\alpha_1...\alpha_n$
Since $\mathcal{M}_i$ and $\mathcal{M}_e$ are atomic equivalent, $V_{\mathcal{M}_i,\mathcal{G}_g}(\Pi\alpha_1 ... \alpha_n) = 1$ iff $V_{\mathcal{M}_e,g}(\Pi\alpha_1 ... \alpha_n) = 1$, by (i) and (i*). Therefore, for every atomic wff $\Pi\alpha_1...\alpha_n$, $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \Pi\alpha_1...\alpha_n$ iff $\mathcal{M}_e$, $g \vDash_e \Pi\alpha_1...\alpha_n$.

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61. Given the existence of inharmonious intensional models, there is no function from extensional models to atomic equivalent intensional models. Each extensional model instead has a class of atomic equivalent intensional models. An extensional model's class of atomic equivalent intensional models includes, for every relational statement that is false on the extensional model, all the inharmonious models that account for that statement's falsity inharmoniously. Yet intensional models which differ only in how they account for the falsity of certain relational statements will have the same atomic equivalent extensional model. So suppose that in three otherwise identical intensional models, model one has it that **R**[_ , Ben] is part of [Amy] but **R**[Amy, _] isn't part of [Ben]; model two has it that **R**[_ , Ben] isn't part of [Amy] and **R**[Amy, _] isn't part of [Ben]; and model three has it that **R**[_ , Ben] isn't part of [Amy] but **R**[Amy, _] is part of [Ben]. The first and third models are inharmonious, but according to the condition for atomic equivalence, all three models share the atomic equivalent extensional model in which ⟨Amy, Ben⟩ isn't in the set R of ordered pairs. Since each intensional model has some atomic equivalent extensional model, the proof for Models-IE shows that each intensional model satisfies all and only the same formulas as some extensional model.

When A is $\alpha = \beta$
Since $\mathcal{M}_i$ and $\mathcal{M}_e$ are atomic and hence name equivalent, $\mathcal{I}_i(\alpha) = \mathcal{I}_i(\beta) \in \mathcal{D}_s$ iff $\mathcal{I}_e(\alpha) = \mathcal{I}_e(\beta) \in \mathcal{D}$. Therefore, $V_{\mathcal{M}_i,\mathcal{G}_g}(\alpha = \beta) = 1$ iff $V_{\mathcal{M}_e,g}(\alpha = \beta) = 1$, by (ii) and (ii*). So for every wff $\alpha = \beta$, $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \alpha = \beta$ iff $\mathcal{M}_e$, $g \vDash_e \alpha = \beta$.

For the inductive hypothesis (IH), assume that for every formula B less complex than formula A below, each $\mathcal{M}_i$ is such that for some atomic equivalent extensional model $\mathcal{M}_e$ and some assignment g, $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i$ B iff $\mathcal{M}_e$, $g \vDash_e$ B. For the inductive step, we show that the same holds for any (non-atomic) wff A, assuming IH. We proceed by cases of A's main operator. (For $\neg\phi$ and $\phi \supset \psi$ each, I will again prove the left-to-right direction only, since the right-to-left direction is symmetrical to it.)

When A is $\neg\phi$
Suppose $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \neg\phi$. Then, $V_{\mathcal{M}_i,\mathcal{G}_g}(\neg\phi) = 1$. So $V_{\mathcal{M}_i,\mathcal{G}_g}(\phi) = 0$, by (iiia). By IH, $V_{\mathcal{M}_e,g}(\phi) = 0$. So $V_{\mathcal{M}_e,g}(\neg\phi) = 1$, by (iiia*), and $\mathcal{M}_e$, $g \vDash_e \neg\phi$.

When A is $\phi \supset \psi$
Suppose $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \phi \supset \psi$. Then, $V_{\mathcal{M}_i,\mathcal{G}_g}(\phi \supset \psi) = 1$. By (iiib), either $V_{\mathcal{M}_i,\mathcal{G}_g}(\phi) = 0$ or $V_{\mathcal{M}_i,\mathcal{G}_g}(\psi) = 1$. By IH, then, either $V_{\mathcal{M}_e,g}(\phi) = 0$ or $V_{\mathcal{M}_e,g}(\psi) = 1$. As a result, $V_{\mathcal{M}_e,g}(\phi \supset \psi) = 1$, by (iiib*). Therefore, $\mathcal{M}_e$, $g \vDash_e \phi \supset \psi$.

When A is $\forall\alpha\phi$
Suppose $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \forall\alpha\phi$. Then, $V_{\mathcal{M}_i,\mathcal{G}}(\forall\alpha\phi) = 1$, and every $[u]$ in $\mathcal{D}_s$ is such that $V_{\mathcal{M}_i,\mathcal{G}^{\alpha}_{[u]}}(\phi) = 1$, by (iiic). The variable interpretation $\mathcal{G}$ is based on the assignment g so that any $\mathcal{G}^{\alpha}$-variants $\mathcal{G}^{\alpha}_{[i]}$ and any α-variants g^{α}_i are such that $\mathcal{G}^{\alpha}_{[i]} = f_2(g^{\alpha}_i)$. Then, by IH, for every $u \in \mathcal{D}$, $V_{\mathcal{M}_e,g^{\alpha}_u}(\phi) = 1$, and, by (iiic*), $V_{\mathcal{M}_e,g}(\forall\alpha\phi) = 1$. So $\mathcal{M}_e$, $g \vDash_e \forall\alpha\phi$.

Suppose $\mathcal{M}_e$, $g \vDash_e \forall\alpha\phi$. Then, $V_{\mathcal{M}_e,g}(\forall\alpha\phi) = 1$ and, by (iiic*), for every $u \in \mathcal{D}$, $V_{\mathcal{M}_e,g^{\alpha}_u}(\phi) = 1$. Given the variable assignment g and each of its α-variants g^{α}_i, there is a variable interpretation $\mathcal{G}$ whose $\mathcal{G}^{\alpha}$-variants are such that $\mathcal{G}^{\alpha}_{[i]} = f_2(g^{\alpha}_i)$. By (IH), then, every $[u]$ in $\mathcal{D}_s$ is such that $V_{\mathcal{M}_i,\mathcal{G}^{\alpha}_{[u]}}(\phi) = 1$. By (iiic), $V_{\mathcal{M}_i,\mathcal{G}}(\forall\alpha\phi) = 1$, and $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \forall\alpha\phi$.

By induction, each intensional model $\mathcal{M}_i$ has some atomic equivalent extensional model $\mathcal{M}_e$ such that for some assignment g, $\mathcal{M}_i$, $\mathcal{G}_g \vDash_i \phi$ iff $\mathcal{M}_e$, $g \vDash_e \phi$. This establishes Models-IE.

A.3 Proof of Equivalence
With Models-IE and Models-EI in hand, we may prove Equivalence. First, we'll prove the left-to-right direction, i.e., if $\Gamma \vDash_e \phi$, then $\Gamma \vDash_i \phi$. For reductio, assume that $\Gamma \vDash_e \phi$ but $\Gamma \nvDash_i \phi$. This says, first, that every extensional model $\mathcal{M}_e$ which satisfies the wffs in Γ also satisfies ϕ, and, second, that there is at least one intensional model $\mathcal{M}_i$ which satisfies the wffs in Γ but fails to satisfy ϕ. But if there is such an intensional model, then Models-IE implies that there is an extensional model which satisfies the wffs in Γ but fails to satisfy ϕ. This contradicts our assumption that no extensional model satisfies the wffs in Γ but fails to satisfy ϕ. Therefore, we've established the left-to-right direction of Equivalence.

Next, we prove the right-to-left direction, i.e., if $\Gamma \vDash_i \phi$, then $\Gamma \vDash_e \phi$. For reductio, assume that $\Gamma \vDash_i \phi$ but $\Gamma \nvDash_e \phi$. This says, first, that every intensional model $\mathcal{M}_i$ which satisfies the wffs in Γ satisfies ϕ, and, second, that there is at least one extensional model $\mathcal{M}_e$ which satisfies the wffs in Γ but not ϕ. But given such an extensional model, Models-EI implies that there is an intensional model which satisfies the wffs in Γ but fails to satisfy ϕ. And this contradicts our assumption that there is no such intensional model. So we have also proved the right-to-left direction of Equivalence, which completes the proof for Equivalence itself.

To review, my intensional approach provides an account of logical consequence which is extensionally equivalent to the account of logical consequence in a standard extensional approach. As a result, it is relatively easy to show that my intensional approach captures the strong soundness and completeness of a standard deductive system for classical first-order logic.[62]

List of Abbreviations

A = G.W. Leibniz. 1923–. *Gottfried Wilhelm Leibniz: Sämtliche Schriften und Briefe*, ed. Deutsche Akademie der Wissenschaften (Darmstadt and Berlin: Akademie-Verlag). Cited by series, volume, and page.

AG = G.W. Leibniz. 1989. *Philosophical Essays*, ed. and tr. R. Ariew and D. Garber (Indianapolis: Hackett).

C = G.W. Leibniz. 1975. *Opuscules et Fragments inédits de Leibniz*, ed. L. Couturat (Paris: Durand, 1857; reprinted Hildesheim: Olms).

DM = G.W. Leibniz. 1686. *Discourse on Metaphysics* in G IV 427–463. Cited by section.

G = G.W. Leibniz. 1875–1890. *Die Philosophische Schriften von Gottfried Wilhelm Leibniz*, ed. C.I. Gerhardt (Berlin: Weidmann; reprinted Hildesheim: Olms, 1960). Cited by volume and page.

L = G.W. Leibniz. 1969. *Philosophical Papers and Letters*, ed. and tr. L.E. Loemker, 2nd ed. (Dordrecht: Reidel).

LA = G.W. Leibniz. 2016. *The Leibniz-Arnauld Correspondence*, ed. and trans. S. Voss (New Haven: Yale University Press). Cited by page number.

62. Strong soundness with respect to an extensional approach says that if ϕ is provable in first-order logic from a set of wffs Γ, then ϕ is an extensional logical consequence of the wffs in Γ (i.e., if $\Gamma \vdash \phi$, then $\Gamma \vDash_e \phi$). And strong completeness with respect to an extensional approach says that if ϕ is an extensional logical consequence of the wffs in Γ, then ϕ is provable in first-order logic from the wffs in Γ (i.e., if $\Gamma \vDash_e \phi$, then $\Gamma \vdash \phi$). So the left-to-right half of Equivalence and the standard soundness result imply that my intensional approach is strongly sound. The right-to-left half of Equivalence and the standard completeness result imply that my intensional approach captures strong completeness. Thanks to David Buller for guidance on this proof.

NE = G.W. Leibniz. 1996. *New Essays on Human Understanding*, tr. and ed. P. Remnant and J. Bennett (Cambridge: Cambridge University Press). Cited by book, chapter, section, and page.

P = G.W. Leibniz. 1966. *Leibniz: Logical Papers*, ed. and trans. G. H. R. Parkinson (Oxford: Oxford University Press). Cited by page.

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