MODAL INTENSIONALISM*

You often hear that our world is one among many possible worlds. Although philosophers disagree about what possible worlds are, many agree about how to use them. Take David Lewis and the so-called ersatzers, for example. Lewis believes that possible worlds are spatio-temporally and causally isolated island universes.[1] Any way things could be is the way some island universe really is. Ersatzers reject Lewis's concrete universes and instead tie possible worlds to abstract representations of ways the world might have been. Following Lewis, I will call an abstract object (or collection of abstract objects) a *world-surrogate* if it represents a complete way things might have been.[2] Candidates for world-surrogacy include suitably large sets of propositions, properties, and states of affairs.[3] Though Lewis and the ersatzers disagree about the nature of possible worlds, they agree that propositions are necessarily true when true in all possible worlds and possibly true when true in at least one.

Whatever possible worlds are, we often do not use them to explain why things must be a certain way. Instead of saying that any mammal must be an animal because, in every possible world, every mammal is also an animal, we might say that any mammal must be an animal because being an animal is part of being mammalian. We seem to think that nothing could exemplify the whole property without

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[1] David Lewis, *On the Plurality of Worlds* (Malden, MA: Blackwell, 1986).

[2] *Ibid.*, p. 137.

[3] See, for example, Robert Merrihew Adams, "Theories of Actuality," *Noûs*, VIII (1974): 211–31; Robert Stalnaker, "Possible Worlds," *Noûs*, X (1976): 65–75; and Alvin Plantinga, *The Nature of Necessity* (New York: Oxford University Press, 1974). In *Mere Possibilities* (Princeton: Princeton University Press, 2011), pp. 9–10, Stalnaker denies that the properties he associates with possible worlds are representations since they are not linguistic or mental entities. Yet in *On the Plurality of Worlds*, *op. cit.*, p. 137, Lewis calls ersatz worlds "representations" partly because it makes sense to speak of what is the case according to them. Stalnaker also assigns this role to possible worlds, so I follow Lewis here.

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exemplifying its parts, and we associate the parts with preconditions for exemplifying the whole.[4] An entirely new semantics for modal logic results when we take this way of speaking seriously. Instead of treating necessary truths as truths in every possible world, we may treat them as the preconditions for the existence of any world at all. Here I develop this alternative semantics for modal propositional logic. The semantics is not Kripkean in any way: neither possible worlds nor accessibility relations nor formally similar stand-ins of either appear in its machinery.

Here is a preview of the new semantics. Our world is alpha, and the property of being alpha accounts for truth. A proposition φ is true just in case the property of being such that φ is part of being alpha.[5] For example, being such that Caesar crosses the Rubicon is part of being alpha. So 'Caesar crosses the Rubicon' is true. But now consider the property of being such that $2 + 2 = 4$. It is not only part of being alpha but also part of being a world in general, the property which accounts for necessary truth. A proposition φ is necessarily true just in case *being such that φ* is part of *being a world*. The parts of being a world correspond to preconditions for anything's being a world in general.

My semantics requires properties and a property parthood relation. I discuss properties in section III and property parthood in sections IV and V.[6] After I unveil the semantics in section VI, I show in section VII how property parthood principles validate axioms of the normal modal propositional calculi. In section IX, I argue that, compared to possible worlds semantics, the meanings in the new semantics operate on a more basic level of modal reality, both metaphysically and epistemologically. In the next two sections, I highlight a feature embedded in the formalism of possible worlds semantics and show how embedding another feature yields the non-Kripkean semantics.

I

My approach and the possible worlds approach treat modal space differently. To bring out the difference, consider the sentence 'Fred

[4] Leibniz presents a similar view of the part-whole structure of concepts in *Opuscules et Fragments inédits de Leibniz*, ed. Louise Couturat (Paris: Félix Alcan, 1903), reprint, (Hildesheim: George Olms, 1966), pp. 53, 55; see the translation in *Leibniz: Logical Papers*, ed. G. H. R. Parkinson (New York: Oxford University Press, 1966), pp. 20, 22. Further references to Couturat and Parkinson will follow the pattern here of using C 53, 55/P 20, 22 for the above references. Leibniz associates a concept's parts with its "requisites," and claims that the concept of being metal is part of the concept of being gold because all the requisites of being metal are among the requisites of being gold.

[5] I will ignore the standard markers for use and mention when the meaning is clear.

[6] My property mereology is similar in spirit to L. A. Paul's mereology in "Logical Parts," *Noûs*, XXXVI (2002): 578–96, and L. A. Paul, "Coincidence as Overlap," *Noûs*, XL (2006): 623–59. I compare and contrast Paul's property mereology with my own in section IV.

is tall'. Standard treatments say the sentence is true when Fred is in the extension of 'is tall', the class or set of things which are tall. They treat the truth-value of a subject-predicate sentence as if it depends on whether the subject's referent is a member of the predicate's extension. Membership in a set or class is an *extensional inclusion* relation.[7] So call the approach to logic which treats a sentence's truth-value as depending on extensional inclusion relations *logical extensionalism.*

The traditional alternative to logical extensionalism appeals to a relation between intensional entities. Intensional entities also have extensions in the actual world, but different intensional entities may have the very same extension. The property of being a cordate and the property of being a renate are two examples of an intensional entity.[8] Although the class of cordates is identical to the class of renates (because they have the same members), the property of being a cordate differs from the property of being a renate.

Terms like 'Fred' or 'H_2O' may have two kinds of associated intensional entities. Usually, *fine-grained intensions* are identified with concepts. Many distinguish the fine-grained intension of 'water' from the fine-grained intension of 'H_2O' because we can imagine watery stuff which is not composed of H_2O. Usually, *coarse-grained intensions* are identified with properties. It is commonplace now to identify the property of being water with the property of being H_2O, the coarse-grained intensions of 'water' and 'H_2O', respectively.

A few stipulations will simplify the presentation of the alternative to logical extensionalism. First, I will frame the ensuing discussion of intensional entities in terms of "properties," but I will leave open how fine- or coarse-grained they are, at least insofar as my approach allows. Second, I endorse an abundant conception of properties, which I will explain in section IV. Finally, I assume that predicates express properties. So I will say that the predicate 'is tall', for example, expresses the property of being tall.

The alternative to logical extensionalism does not treat a sentence's truth-value as depending on extensional inclusion relations but instead on whether the subject's intension, a property, includes the property that the predicate expresses. I will call the inclusion relation between properties *intensional inclusion.* On this approach, the sentence 'Fred is tall' is true just in case the property of being Fred includes the

[7] I borrow "extensional inclusion" and "intensional inclusion" from Chris Swoyer, "Leibniz on Intension and Extension," *Noûs*, XXIX (1995): 96–114.

[8] W. V. O. Quine, "Two Dogmas of Empiricism," *Philosophical Review*, LX (1951): 20–43, at p. 21.

property of being tall.[9] For the obvious reason, I will call the approach to logic which treats a sentence's truth-value as depending on intensional inclusion relations *logical intensionalism*. The crucial difference between logical extensionalism and logical intensionalism is the direction of their respective inclusion relations. Whereas the set of tall things includes Fred on the extensionalist approach, *being Fred* includes *being tall* on the intensional approach. Leibniz knew of the inverse relationship between the approaches and famously favored an intensional approach.[10]

I have defined intensional approaches in terms of intensional inclusion. But it is often more natural to speak of a property's parts rather than the properties a property includes. Since property inclusion is the converse of property parthood (being F is part of being G just in case being G includes being F), nothing is lost if we construe intensional approaches in terms of property parthood. For most of what follows, I will opt for property parthood, which I discuss in sections IV and V. In the next section, I show how a new kind of semantics results when we replace the extensional inclusion relation embedded within possible worlds semantics with an intensional inclusion relation.

II

A semantics for a modal logic may accomplish one or two overarching tasks. Minimally, a semantics for modal logic uses a set-theoretical construction called a *model structure* to determine the truth conditions for formulas, including formulas with modal operators like $\Box$ and $\Diamond$. On the basis of those truth conditions, a model structure determines which formulas are valid in which systems. But a semantics may do this and leave the modal operators uninterpreted, disconnected from any modal notions. Alvin Plantinga aptly calls such a semantics *pure*.[11] Kripke semantics is pure and does not have any explicit connection to the modal notions of necessity and possibility.[12] A model structure in

[9] One difficulty for logical intensionalism is the treatment of relational predicates. For a discussion of this difficulty, see G. H. R. Parkinson, *Logic and Reality in Leibniz's Metaphysics* (New York: Oxford University Press, 1965), pp. 39–52. I do not resolve that problem here because I aim to develop an approach to modal propositional logic that contains no relational predicates and so does not require a treatment of relational sentences. In my unpublished paper "Logical Intensionalism," I develop an approach to first-order logic that treats *n*-ary predicates as if they expressed *n* monadic properties with built-in relational aspects. For example, the formula Rab is true just in case (i) *being an x such that Rxb* is part of *being a*, and (ii) *being an x such that Rax* is part of *being b*.

[10] See C 53/P 20.

[11] Plantinga, *Nature of Necessity*, *op. cit.*, p. 127.

[12] Saul Kripke, "A Completeness Theorem in Modal Logic," *Journal of Symbolic Logic*, XXIV (1959): 1–14; and Saul Kripke, "Semantical Analysis of Modal Logic I: Normal Modal Propositional Calculi," *Zeitschrift fur Mathematische Logik und Grundlagen der Mathematik*, IX (1963): 67–96.

Kripke semantics consists of an ordered triple ⟨G, K, R⟩, where G is a member of K and R is a binary relation defined over the members of K. I will assume some familiarity with how these function in Kripke semantics.

One may also intend an interpretation of the model structure to model some portion of modal discourse. Possible worlds semantics co-opts the formalism of Kripke semantics and explicitly associates the members of K with possible worlds (or world-surrogates), G with the actual world, and R with an accessibility relation defined over those possible worlds. One may use possible worlds semantics to model metaphysical, deontic, epistemic, or other kinds of modality with different readings of the modal operators. For example, the box may read "it is obligatory that…" in deontic logic or "it is known that…" in epistemic logic. The possible worlds approach to metaphysical modality says that $\Box\varphi$ (read "necessarily, φ") is true at a world just in case φ is true in every possible world accessible from it and that $\Diamond\varphi$ (read "possibly, φ") is true at a world just in case φ is true in some accessible possible world.

My approach and the possible worlds approach to metaphysical modality differ in their intended model structures—what I will henceforth call *frames*.[13] The frames differ in whether the embedded relation between actuality and worldhood is an intensional or extensional inclusion relation. The possible worlds approach to metaphysical modality involves an extensional inclusion relation since K, the set of possible worlds, includes the actual world, G. Call the possible worlds approach to metaphysical modality where the actual world is a member of a certain class of possible worlds *modal extensionalism*.

The alternative inverts the relationship between actuality and worldhood with an intensional inclusion relation. Call the actual world *alpha*. An intensional frame consists of an ordered triple, ⟨$\mathbb{A}$, $\mathbb{W}$, $\mathbb{P}$⟩, where $\mathbb{A}$ is the property of being alpha, $\mathbb{W}$ is the property of being a world, and $\mathbb{P}$ is a parthood relation defined over properties. *Modal intensionalism* is the semantic approach to metaphysical modality according to which *being a world* is part of *being alpha*. The diagram below depicts the two types of frames:

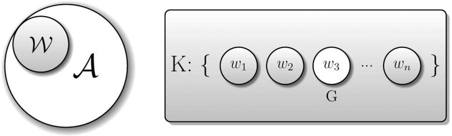

Figure 1. Modal intensionalism (left) and modal extensionalism (right).

[13] A "frame" in possible worlds semantics is usually an ordered pair of a set of worlds and an accessibility relation without any world pegged as actual.

The modal extensionalist's frame is the modal intensionalist's turned "inside out," and this inversion unsurprisingly yields different treatments of the modal notions. Whereas modal extensionalism says that a proposition φ is necessarily true when it is true in every possible world, modal intensionalism says that a proposition φ is necessarily true when *being such that φ* is part of *being a world*.

In what follows, I explain how modal intensionalism works for modal propositional logic. Since, among the standard modal systems, many believe the S5 system captures metaphysical modality best, I will interpret the box as "it is metaphysically necessary that…" and provide an applied semantics for the S5 system of modal propositional logic. The semantics requires that properties have parts in some sense, and I explain this sense of property parthood in the next two sections.

III

When I say that *properties* have parts, I have in mind an abundant and not a sparse conception of properties. According to Lewis, sparse properties account for

> …qualitative similarity, they carve at the joints, they are intrinsic, they are highly specific, the sets of their instances are *ipso facto* not entirely miscellaneous, there are only just enough of them to characterise things completely and without redundancy.[14]

On the abundant conception, however, there is a property corresponding to every predicate.[15] The predicate 'is red or blue' expresses the disjunctive property of being red or blue, and 'is five miles from New York' expresses the extrinsic property of being five miles from New York. When two things share the disjunctive property of being red or blue, for instance, they may do so because one is red and the other blue. Sharing an abundant property does not guarantee qualitative similarity.

The abundant conception licenses more than just disjunctive and extrinsic properties. We say things like "three is not even" and "three is odd and prime" using the predicates 'is not even' and 'is odd and prime'. So on the abundant conception, there are negative properties like *being not-even* and conjunctive properties such as *being odd and prime*. We also sometimes refer to inconsistent properties which nothing could exemplify, like *being round and square* or *being a set of all sets that are not members of themselves*.[16] The abundant conception licenses their existence, too.

[14] Lewis, *On the Plurality of Worlds, op. cit.*, p. 60.

[15] *Ibid.*, pp. 59–60.

[16] By similar reasoning, one may conclude that there is a property of being non-self-exemplified. But does it exemplify itself? If it does, then it would seem to have

Besides disjunctive, conjunctive, negative, and inconsistent properties, the abundant conception also countenances propositional properties. We can also prefix 'is such that' to any sentence to form a predicate that expresses a propositional property.[17] Prefix 'is such that' to 'three is odd' and the result, 'is such that three is odd', expresses the propositional property *being such that three is odd*. There are mixtures, too: disjunctive properties whose disjuncts are propositional properties, conjunctive properties comprised of negative properties, and so on.

Abundant properties abound, to be sure. Those with predilections for desert landscapes may wonder whether the abundant conception countenances too many properties. I accept abundant properties because they play such a useful role in intensional semantic approaches to logic and modality, which are preferable in some ways to their extensional counterparts.[18] For me, then, whether the price of abundant properties is worth their plunder hinges on, first, how metaphysically controversial they are, and, second, how successful intensional approaches to logic and modality are.

On the first point concerning abundant properties, I will remain as metaphysically neutral about their metaphysics as the approach allows. This is only fair. Modal extensionalism as such requires possible worlds or world-surrogates, but these might be spatio-temporally isolated universes, sets of propositions, states of affairs, structural universals, fictions, or something else. Similarly, modal intensionalism as such requires abundant properties like *being a world*, and these properties might be universals, concepts, fictions, or something else. I presently aim to display the loosely interpreted formal features of modal intensionalism, similar to the way one might explain the loosely interpreted ins and outs of possible worlds semantics. Adopting

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the property of non-self-exemplification and so not exemplify itself. But if it does not exemplify itself, it would seem to have the property of non-self-exemplification and so exemplify itself. In "A Theory of Properties," in Dean Zimmerman, ed., *Oxford Studies in Metaphysics, Volume 1* (New York: Oxford University Press, 2004), pp. 107–38, at pp. 135–36, Peter van Inwagen observes a similar problem with his commitment to properties as assertibles. Van Inwagen prefers to deny that there is any such problematic property but does not endorse any particular strategy for doing so. I would prefer to accept the problematic property, but I have no strategy for avoiding paradox. Thanks to Joshua Rasmussen for discussion on this point.

[17] In section VII, I discuss the differences between propositional properties and non-propositional properties in more detail.

[18] I develop an intensional approach to first-order logic in my unpublished paper "Logical Intensionalism." In "New Work for a Theory of Universals," reprinted in *Papers in Metaphysics and Epistemology* (New York: Cambridge University Press, 1999), pp. 8–55, at pp. 16–19, Lewis argues that abundant properties are necessary for sketching a systematic semantics and for characterizing our intentional attitudes.

a sophisticated account of abundant properties would distract from that aim.

Yet I cannot be fully neutral about the metaphysics of abundant properties. Their abundance alone precludes what I will call *Aristotelianism*, the view that properties are object-dependent and do not exist unless they are exemplified by an object. Aristotelianism implies that there are no unexemplified properties. *Platonism* denies that properties are object-dependent and thus permits unexemplified properties. I endorse Platonism about abundant properties and accept properties that never punch their tickets as properties *of* something. Aristotelianism may cohere with some form of modal intensionalism, but the approach runs more smoothly with Platonism's larger stockpile.

IV

When I say that properties have *parts*, I have in mind a conception of parthood that captures many ordinary parthood judgments. We say that being an animal is part of being a mammal and that being red is part of being crimson, for instance.[19] This may be nothing more than a loose and popular way of speaking. Or it may be worth taking more seriously.

I suspect we talk this way because we have some meaningful conception of property parthood. A parthood judgment of the form *being F* is part of *being G* seems to imply that whatever is G is thereby also F. When we say that *being F* is part of *being G* we might mean that being F is "part of what it takes" to be G.[20] Sure enough, whatever is crimson is thereby red, and being red is part of what it takes to be crimson. We will have to leave it at that, however. Although I will not offer an analysis of property parthood, I will propose some axioms for the relations that implicitly define it.[21]

Even so, the following modal analysis of property parthood is tempting: *being F* is part of *being G* if and only if, necessarily, whatever is G is also F. This analysis captures many of our ordinary judgments about property parthood. For example, being red is part of being crimson; necessarily, whatever is crimson is red. But I reject the modal analysis because I want to leave open whether necessarily coextensive properties

[19] For similar examples, see Achille Varzi, "Mereology," *Stanford Encyclopedia of Philosophy* (2009), section 1; and David M. Armstrong, *A Theory of Universals*, Volume 2 of *Universals and Scientific Realism* (Cambridge, UK: Cambridge University Press, 1978), pp. 37–38.

[20] Recently, Gideon Rosen has used the "part of what it takes" locution while discussing the grounding relation. See his paper "Metaphysical Dependence: Grounding and Reduction," in Bob Hale and Aviv Hoffman, eds., *Modality: Metaphysics, Logic, and Epistemology* (New York: Oxford University Press, 2010), pp. 109–36.

[21] I provide an analysis of property parthood in my unpublished paper "Modal Idealism."

such as *being triangular* and *being trilateral* are distinct. The modal analysis implies that necessarily coextensive properties are identical. Necessarily, whatever is triangular is also trilateral, so the modal analysis implies that *being trilateral* is part of *being triangular*. By similar reasoning, we may conclude that *being triangular* is part of *being trilateral*. If *being trilateral* and *being triangular* are parts of each other, we would have to conclude that *being trilateral* and *being triangular* are the same property, given the usual definition of parthood.[22] The modal analysis precludes necessarily coextensive though distinct properties. Modal intensionalism defines modal notions in terms of property parthood and therefore permits necessarily coextensive though distinct properties.

The most well-known philosophical conception of property parthood occurs in discussions about universals, properties which are wholly located wherever they are exemplified. These discussions focus specifically on structural universals. The *structure* of a structural universal roughly includes whatever universals a thing's spatio-temporal parts exemplify in virtue of exemplifying that universal.[23] The *structural conception* of parthood says that a universal's parts are those universals that appear in its structure. An H_2O molecule is composed of two hydrogen atoms and an oxygen atom. So the universals *being hydrogen* and *being oxygen* appear in the structure of *being an* H_2O *molecule* and are therefore parts of *being an* H_2O *molecule*.

On the structural conception, Lewis worries whether *being hydrogen* is part of *being an* H_2O *molecule* once or twice.[24] On the one hand, *being hydrogen* is a universal that is wholly located in all its instances, so the answer seems to be "once." But this reasoning threatens to erase the structural differences of molecules that share the same kinds but not the same number of atoms, like H_2O and H_2O_2. On the other hand, *being hydrogen* appears twice in *being an* H_2O *molecule*'s structure, so it seems to be a part of H_2O twice over. But what are there two of, since *being hydrogen* is wholly present in each of its instances? The structural conception seems incoherent.

My preferred conception of parthood evades Lewis's argument because my conception disagrees with the structural conception about which properties are parts of which. *Being hydrogen* is part of *being an* H_2O *molecule* on the structural conception but not on mine. Being hydrogen is not part of what it takes to be an H_2O molecule. If it were, H_2O molecules would be hydrogen atoms. So there is no question

[22] See Definition 1 below. Thanks to an anonymous referee for this point.

[23] David Lewis, "Against Structural Universals," *Australasian Journal of Philosophy*, LXIV (1986): 25–46, at p. 33.

[24] *Ibid.*

about whether *being hydrogen* is part of *being an H_2O molecule* once or twice. However, *having a hydrogen atom as a spatio-temporal part* may very well be part of *being an H_2O molecule* on my conception. Hydrogen atoms are spatio-temporal parts of H_2O molecules. Since an H_2O molecule has two hydrogen atoms as spatio-temporal parts, we may rephrase Lewis's original worry: is *having a hydrogen atom as a spatio-temporal part* part of *being an H_2O molecule* once or twice?

My conception of property parthood survives this incarnation of Lewis's worry because Platonistic abundant properties are not wholly present in their instances. Consider a specific H_2O molecule, Harry, whose two hydrogen atoms are Matthew and Carl.[25] *Having a hydrogen atom as a spatio-temporal part* is part of *having Matthew as a spatio-temporal part* and *having Carl as a spatio-temporal part.* These latter two properties overlap. Furthermore, *having Matthew as a spatio-temporal part* and *having Carl as a spatio-temporal part* are both parts of *being Harry*. Given the transitivity of parthood, *having a hydrogen atom as a spatio-temporal part* is part of *being Harry* once. What is "twice" is not how many times *having a hydrogen atom as a spatio-temporal part* is part of *being Harry*, but rather how many properties of having this or that particular hydrogen atom as a spatio-temporal part are part of *being Harry* and also overlap with respect to *having a hydrogen atom as a spatio-temporal part.*[26] The situation resembles cases of overlapping physical objects. If object x has y and z as proper parts, and y and z themselves overlap with respect to w, then we may infer that w is part of x via its being part of y or via its being part of z. The two inference chains signify how many of x's proper parts (besides w) overlap with respect to w, not how many times w is part of x.

Given the intelligibility of property parthood, it will be beneficial to compare and contrast available conceptions of property parthood

[25] For two other recent responses, see Katherine Hawley, "Mereology, Modality and Magic," *Australasian Journal of Philosophy*, LXXXVIII (2010): 117–33; and Karen Bennett, "Having a Part Twice Over," *Australasian Journal of Philosophy*, XCI (2013): 83–103.

[26] I suspect Lewis would agree with something like this response. In "Against Structural Universals," *op. cit.*, p. 41, n. 21, Lewis mentions briefly that there is room for a mereology of conjunctive universals, where the conjuncts are present wherever the conjunction is. He says that this picture is "quite natural" and that "he has no quarrel" with any of it. Though my properties are not universals, my preferred conception is amenable to a conjunctive analysis. If you want, we can say that being F is part of being G just in case being G is a conjunction of properties, one of which is being F. My preferred conception owes much to Leibniz's mereological view of concepts. In P 135, Leibniz endorses the biconditional that the concept F is identical to the conjunction of F and G if and only if the conjunctive concept F contains G.

with my own.[27] My conception is similar in spirit to L. A. Paul's.[28] We each construct a property mereology and then implement it in metaphysically interesting ways. The major differences are the kinds of properties we implement and how we implement them. Paul endorses a "relatively sparse" Aristotelian conception of properties and uses her property mereology in a bundle theory of material objects.[29] I endorse a Platonistic conception of abundant properties and use my property mereology in an account of modality.

My mereology and Paul's are formally similar, but our differences in approach lead to three further differences. First, due to the types of properties each mereology governs, I endorse unrestricted composition, and Paul does not.[30] Second, my mereology contains two additional axioms that govern properties of properties. Finally, my mereology is meant to capture a way we often speak about properties; Paul's is not. So whereas my account says that *being mammalian* is part of *being a dog* and seems to imply that determinables are parts of their determinates generally, Paul's account permits the view that determinates are parts of their determinables.

On the *Boolean conception* of parthood, properties compose other properties through analogues of the Boolean operations of conjunction, disjunction, and negation. According to Dean Zimmerman, we can reach a property's parts by "successive eliminations of disjuncts, conjuncts, and the ontological analogue of the negation operator."[31] Gary Rosenkrants and Joshua Hoffman endorse a similar conception according to which "a conjunctive property has each of its conjuncts as a logical part, a disjunctive property has each of its disjuncts as a logical part, and so forth."[32] The Boolean conception implies that *being red* is part of *being not-red*. This is not something we would ordinarily say, not in the same sense of 'part' that we would say *being a mammal* is part of *being a dog*. We would not ordinarily say that *being red* is part of *being not-red* because we know that red things are not not-red.

My focus is the conception of parthood that would account for many of our parthood judgments, including the judgment that *being*

[27] My conception of property parthood, and the way I use it to account for modal claims, bears important similarities to Tony Roy's views defended in "Worlds and Modality," *Philosophical Review*, CII (1993): 335–61. I came across this wonderful paper much too late in the process to engage with it here.

[28] See Paul, "Logical Parts," *op. cit.*, and "Coincidence as Overlap," *op. cit.*

[29] Paul, "Coincidence as Overlap," *op. cit.*, p. 634.

[30] *Ibid.*

[31] Dean Zimmerman, "Immanent Causation," *Philosophical Perspectives*, XI (1997): 433–71, at p. 463.

[32] Gary Rosenkrantz and Joshua Hoffman, "The Independence Criterion of Substance," *Philosophy and Phenomenological Research*, V (1991): 835–53, at p. 845.

mammalian is part of *being a dog*. But the set-theoretic account of properties, insofar as it can say that properties are parts of properties, says that *being a dog* is part of *being mammalian*.[33] On the set-theoretic account, each property is a set of possible individuals: *being mammalian* is the set of mammals in any possible world, and *being a dog* is the set of dogs in any possible world. Because whatever is a dog is thereby a mammal and not vice versa, the set of possible dogs is a subset of the set of possible mammals. So if one set is part of the other, the set which is the property of being a dog is part of the set which is the property of being mammalian, not the other way around.[34] For similar reasons, my preferred conception also conflicts with the view that properties are functions from possible worlds to individuals.

We often speak as if properties have parts. We say things like "*being red* is part of *being crimson*" and "*being an animal* is part of *being* mammalian." None of the available conceptions of property parthood explicitly captures these sorts of judgments. Nor were they meant to. But if properties have parts in the way we often say they do, we can leverage an understanding of our parthood judgments to propose a formal theory of property parthood.

V

Modal intensionalism defines modal notions in terms of property parthood. Various mereological principles inspire the restrictions on frames that validate various modal axioms. Below, I sketch a formal theory of property parthood.

Let us begin with definitions. In this section, lower-case variables (for example, *x*, *y*, *z*) range over properties instead of objects.[35] And 'proper part' is the chosen primitive in terms of which I will define other mereological notions.

Definition 1. *x* is *part* of *y* iff *x* is a proper part of *y* or identical to *y*.
Definition 2. *x* and *y* *overlap* iff there is some *z* which is part of both *x* and *y*.

It will be beneficial to detour through some examples. *Being mammalian* is part of *being a cat*. But if *being a cat* were part of *being mammalian*, all mammals would be cats. Some mammals are not cats, of course, so *being a cat* is not part of *being mammalian*. Thus, *being mammalian* and *being a cat* are not identical, by the non-identity of discernibles. *Being mammalian* is part of but not identical to *being a cat*, so by Definition 1,

[33] Lewis, *On the Plurality of Worlds*, *op. cit.*, p. 60.
[34] Lewis claims that the subclasses of a class are its parts in *Parts of Classes* (Cambridge, MA: Basil Blackwell, 1991), pp. 3–5.
[35] However, I will sometimes use upper-case letters to highlight the link between a predicate 'F' and the property of being F.

being mammalian is a proper part of *being a cat*. For similar reasons, *being mammalian* is a proper part of *being a dog*, so the properties of being a cat and being a dog overlap.

Using these definitions, we define two other notions:

Definition 3. *x* and *y* are *disjoint* iff they do not overlap.
Definition 4. *x* is a *sum* of the *y*s iff (each of the *y*s is part of *x* and (any *z* overlaps one of the *y*s iff *z* overlaps *x*)).

Since Definitions 3 and 4 do not figure importantly in the modal semantics, and since they are straightforward analogues of definitions in classical extensional mereology, I will take their meaning for granted and move on to the axioms.

The first axiom says that proper parthood is asymmetric:

Asymmetry. If *x* is a proper part of *y*, *y* is not a proper part of *x*.

The guiding idea is that properties subsume and outstrip their proper parts, and subsuming while outstripping is asymmetric. Consider the properties *being mammalian* and *being a cat*. *Being mammalian* is a proper part of *being a cat*—there is more to being a cat than to being a mammal. But *being a cat* is not a proper part of *being mammalian*. If it were, we could infer from Definition 1 that *being a cat* is part of *being mammalian*, which is false. So *being a cat* is not a proper part of *being mammalian* either.

The second axiom says that proper parthood is transitive:

Transitivity. If *x* is a proper part of *y* and *y* is a proper part of *z*, then *x* is a proper part of *z*.

The trio of properties *being an animal*, *being mammalian*, and *being a cat* nicely illustrates the principle. *Being an animal* is a proper part of *being mammalian*, and *being mammalian* is a proper part of *being a cat*. By Transitivity, we correctly infer that *being an animal* is a proper part of *being a cat*.

Now let us consider a principle whose axiomatic status is not as clear:

Weak Supplementation. If *x* is a proper part of *y*, then *y* has another proper part disjoint from *x*.

Weak Supplementation rules out the scenario in which a property has a single proper part, as well as the scenario in which a property's proper parts all overlap one another. On the modal analysis of parthood (*being F* is part of *being G* if and only if, necessarily, whatever is G is F) Weak Supplementation fails universally. Given any two properties *being F* and *being G*, there is a disjunctive property of being F or G. Since, necessarily, whatever is F is also F or G, and, necessarily, whatever is G is also

F or G, the modal analysis implies that *being F or G* is part of both *being F* and *being G*. Indeed, the modal analysis implies that for any group of properties whatsoever, there is some disjunctive property that is part of them all. I would be happy to avoid the complications that result from denying Weak Supplementation while simultaneously trying to save the intuition that subtracting a property's proper part should leave some remainder.[36] Since I reject the modal analysis for independent reasons (see section IV), I have one less reason to deny Weak Supplementation. There may be other reasons to deny Weak Supplementation, but I suspect that denying it would have little influence on the modal semantics as I develop it.

The following principle is also disputable:

> General Sum Principle. For any specifiable set of properties whatever, there is a sum of those properties.

Two properties *preclude* each other when one property is the negative correlate of the other, like *being not-F* and *being F*, and a property is *inconsistent* when two of its parts preclude each other. The General Sum Principle posits inconsistent properties, which some may find worrisome. But we are no worse off given the Platonistic view of abundant properties endorsed in section III. The abundant conception says the predicate 'is F and not-F' expresses a property, and the General Sum Principle says there is a sum composed of the properties *being F* and *being not-F*. The sums secured by the General Sum principle are the properties licensed by an abundant conception of Platonistic properties. So the General Sum Principle commits the Platonist about abundant properties to no more properties than she already was.

The abundant conception also licenses properties of properties, or *meta-properties*. Consider *being the property of being a dog*, for example, which is distinct from the property of being a dog. Meta-properties have parts, too, and the final two axioms concern their parts. Here is the first axiom for meta-properties:

> Inclusivity. If *x* is part of *y*, then the property of having *x* as a part is itself part of the property of being *y*.[37]

Being mammalian is part of *being a dog*. And what it is to be the property of being a dog includes its having being mammalian as a part.

[36] See Varzi, "Mereology," *op. cit.*, sec. 3.1.

[37] It is crucial to distinguish *y*, which is a property, from the property of being *y*. Dogs exemplify the property of being a dog. They do not exemplify the property of being the property of being a dog. The property of being a dog exemplifies the property of being the property of being a dog.

Having the property of being mammalian as a part is part of *being the property of being a dog*. When a meta-property is the property of being some property, I will call the latter the *base property*. Given the base property of being a dog, there is a meta-property of being the property of being a dog. Inclusivity says that some parts of meta-properties concern which properties are parts of their base properties. Being a base property or meta-property is a relative matter, for we can treat any meta-property as a base property. For instance, we can treat *being the property of being a dog* as the base property of *being the property of being the property of being a dog*.

Here is the last of our axioms:

> Exclusivity. If x is not part of y, then the property of not having x as a part is itself part of the property of being y.

Exclusivity says that if a property is not part of another property, the latter's not having that property as a part is part of being that latter property. That is, some parts of meta-properties concern which properties are not parts of their base properties. *Being a dog*, for example, is not part of *being human*. So the property of not having *being a dog* as a part is itself part of being the property of being human.

VI

Intensional approaches to logic use intensional inclusion relations among properties to determine the assignment of truth-values. But an intensional approach to the modal propositional calculi cannot use properties associated with names or predicates simply because there are no names or predicates in the propositional calculus. So intensional inclusion relations among properties may determine the assignment of truth-values to propositions only if the properties correspond to the unanalyzed propositions themselves.

There is a function from propositions to the relevant properties. Start with the usual stock of proposition letters p_0, p_1, p_2, … , which represent unanalyzed propositions. Modal propositional logic contains non-atomic propositions, too, and those non-atomic propositions require connectives. So throw in the connectives written $\neg$, $\wedge$, $\vee$, and $\supset$, which represent negation, conjunction, disjunction, and the material conditional, respectively. Now let '[…]' be a 1–1 function from propositions to their corresponding propositional properties. For every proposition φ, there is a unique property $[\varphi]$, or *being such that* φ. Given this function, $[\varphi]$ is identical to $[\psi]$ just in case φ is identical to ψ.

There are at least two ways in which propositional properties differ from non-propositional properties. Roughly, when something exemplifies a non-propositional property, the non-propositional property is part of being that thing. For example, when Fred is tall, *being tall* is part of *being Fred*. Propositional properties are more complicated. When alpha exemplifies the propositional property of being such that Fred is tall, for instance, *being such that Fred is tall* is part of *being alpha*. And alpha is such that Fred is tall just in case Fred is tall. Therefore, *being such that Fred is tall* is part of *being alpha* just in case *being tall* is part of *being Fred* (*as Fred actually is*). However, since I have restricted my focus to unanalyzed propositions and their corresponding propositional properties to develop a semantics for modal propositional logic, delving further into the parthood relations among the semantic values of names and predicates would take us too far afield.

Secondly, propositional properties bear a special connection to actuality and truth. On the intensional approach, a proposition φ is true just in case its corresponding property *being such that* φ is part of *being alpha*.[38] So on the intensional approach, there is an equivalence between truth and actuality.[39] A propositional property's being part of *being alpha* and its corresponding proposition's being true are, in a sense, one and the same. Where $\mathbb{A}$ is the property *being alpha* and '<' reads "is part of":

(A) φ is true $=_{\text{df.}}$ $[\varphi] < \mathbb{A}$.

Here is how this treatment "looks":

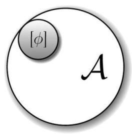

Figure 2. An intensional approach to truth.

Since the S5 system contains classical logic, and since I aim to offer an applied semantics for S5, I will assume the world behaves classically in ways that secure the theorems of classical logic. So I assume that $\mathbb{A}$, the property of being alpha, is complete: for every pair of propositions φ and $\neg\varphi$, at least one of either $[\varphi]$ or $[\neg\varphi]$ is part of $\mathbb{A}$. I also assume that $\mathbb{A}$ is consistent: for every pair of propositions φ and $\neg\varphi$, at most one of either $[\varphi]$ or $[\neg\varphi]$ is part of $\mathbb{A}$. Therefore, if $[\varphi]$ is a part of $\mathbb{A}$, $[\neg\varphi]$ is not, and vice versa. $\mathbb{A}$'s completeness secures the law of excluded

[38] Most think that truth is "extensional" in the sense that when modal operators are not an issue, one may substitute any true proposition for any other *salva veritate*. This is compatible with the present intensional treatment of truth.

[39] This is the intensional side of the "true-story" theory of actuality in Adams, "Theories of Actuality," *op. cit.*, p. 226.

middle and guarantees that there are no truth-value gaps. Its consistency secures the law of non-contradiction and guarantees that there are no true contradictions. One might reject 𝔸's completeness or consistency because of vagueness or liar paradoxes, but I ignore these complications because the first test for an alternative approach to modal logic should be whether and how it handles the classical systems.

A number of auxiliary assumptions seem to follow from our intuitive understanding of '…and…' statements, '…or…' statements, 'if…then…' statements, and the like. For example:

(i) if $[\varphi \supset \psi]$ and $[\varphi]$ are parts of 𝔸, so is $[\psi]$,
(ii) $[\varphi]$ is part of 𝔸 iff $[\neg\neg\varphi]$ is,
(iii) $[\varphi \wedge \psi]$ is part of 𝔸 iff $[\varphi]$ and $[\psi]$ are, and
(iv) $[\varphi \vee \psi]$ is part of 𝔸 iff either $[\varphi]$ or $[\psi]$ is.

These secure theorems and validate various inferences of propositional logic. For example, if $[\varphi \wedge \psi]$ is part of 𝔸, then the conjunction $\varphi \wedge \psi$ is true, by (A). But $[\varphi \wedge \psi]$ is part of 𝔸 if and only if both $[\varphi]$ and $[\psi]$ are, by (iii). So if $[\varphi \wedge \psi]$ is part of 𝔸, then so are $[\varphi]$ and $[\psi]$. If $[\varphi]$ and $[\psi]$ are both parts of 𝔸, then both φ and ψ are true, by (A). Hence, (iii) validates the inferences from a conjunction to its conjuncts.

Now we proceed from truth to necessary truth. Modal extensionalism treats necessary truth as truth in every possible world:

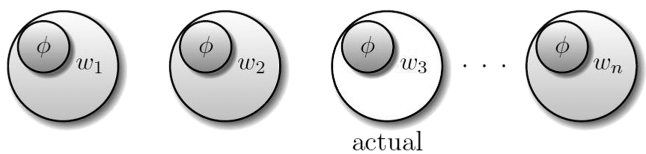

Figure 3. Necessary truth according to modal extensionalism.

Modal intensionalism says that φ is necessarily true if and only if its corresponding propositional property *being such that* φ is part of being a world. More formally, where 𝕎 is the property *being a world*:

(N) $\Box\varphi$ is true $=_{\text{df.}}$ $[\varphi] < \mathbb{W}$.

In the S5 system, we may depict (N) as follows:

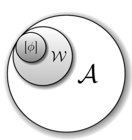

Figure 4. Necessary truth according to modal intensionalism.

I assume (reasonably, I think) that, like the propositional parts of $\mathbb{A}$, the propositional parts of $\mathbb{W}$ behave in ways that help secure our intuitive understanding of '…and…' statements, '…or…' statements, 'if…then…' statements, and so on. This suggests a number of principles that govern how propositional properties behave in $\mathbb{W}$, including:

(v) if $[\varphi \supset \psi]$ and $[\varphi]$ are parts of $\mathbb{W}$, so is $[\psi]$,
(vi) $[\varphi]$ is part of $\mathbb{W}$ iff $[\neg\neg\varphi]$ is,
(vii) $[\varphi \wedge \psi]$ is part of $\mathbb{W}$ iff $[\varphi]$ and $[\psi]$ are, and
(viii) if $[\varphi]$ is part of $\mathbb{W}$, so is $[\varphi \vee \psi]$.[40]

Joined with (N), these principles validate a number of intuitive modal inferences. Consider (vii), for instance. It can be easily shown that (vii) and (N) validate the inferences from $\Box(\varphi \wedge \psi)$ to both $\Box\varphi$ and $\Box\psi$ and from both of these back again to $\Box(\varphi \wedge \psi)$.

These sorts of principles, along with (N), imply that the theorems of propositional logic are necessarily true. For example, given (vii), if $[\varphi \wedge \psi]$ is part of $\mathbb{W}$, $[\varphi]$ is, too. Then, based on our intuitive understanding of 'if…then…', $[(\varphi \wedge \psi) \supset \varphi]$ is also part of $\mathbb{W}$. We then infer from (N) that $(\varphi \wedge \psi) \supset \varphi$ is necessarily true. The property $[(\varphi \wedge \psi) \supset \varphi]$ corresponds to a theorem of propositional logic. Intuitively, all other theorems of propositional logic have corresponding propositional properties that are also parts of $\mathbb{W}$, so they are also necessarily true. These considerations justify an important inference rule in the weakest normal modal system K (a system that has all the theorems of propositional logic as theorems):

Necessitation Rule. If φ is a theorem of K, then so is $\Box\varphi$.

Next we define possibility in terms of necessity. $\neg\varphi$ is necessary just in case φ is impossible. So φ is not impossible—that is, possible—just in case $\neg\varphi$ is not necessary. By (N), $\neg\varphi$ is not necessary when $[\neg\varphi]$ is not part of $\mathbb{W}$. Therefore, a proposition φ is possible when $[\neg\varphi]$ is not part of $\mathbb{W}$:

(P) $\Diamond\varphi$ is true $=_{\text{df.}} [\neg\varphi] \not< \mathbb{W}$

What is possibly true corresponds to what $\mathbb{W}$'s parts do not preclude. Traditionally understood, the necessity and possibility operators are interdefinable: $\Box\varphi$ is equivalent to $\neg\Diamond\neg\varphi$ (and $\neg\Box\neg\varphi$ is equivalent to $\Diamond\varphi$). Modal intensionalism justifies both of these equivalences, but I will concentrate on the first. Suppose, then, that $\neg\Diamond\neg\varphi$ is true.

[40] An analogue of (iv), the stronger biconditional principal that $[\varphi \vee \psi]$ is part of $\mathbb{W}$ iff either $[\varphi]$ or $[\psi]$ is, presumably does not hold for $\mathbb{W}$ because $\mathbb{W}$ is not complete like $\mathbb{A}$ is.

By (P), $\Diamond\neg\varphi$ is true when $[\neg\neg\varphi]$ is not part of $\mathbb{W}$. So $\neg\Diamond\neg\varphi$ is true when $[\neg\neg\varphi]$ is part of $\mathbb{W}$. Given that $[\neg\neg\varphi]$ is part of $\mathbb{W}$, $[\varphi]$ is part of $\mathbb{W}$, too, by principle (vi). As a result, $\Box\varphi$ is true, by (N). Therefore, if $\neg\Diamond\neg\varphi$ is true, then so is $\Box\varphi$. Now suppose that $\Box\varphi$ is true. By (N), $[\varphi]$ is part of $\mathbb{W}$. Again, by principle (vi), since $[\varphi]$ is part of $\mathbb{W}$, $[\neg\neg\varphi]$ is also part of $\mathbb{W}$. And, as before, $\neg\Diamond\neg\varphi$ is true when $[\neg\neg\varphi]$ is part of $\mathbb{W}$. Therefore, if $\Box\varphi$ is true, $\neg\Diamond\neg\varphi$ is true, which completes the proof.

VII

If modal intensionalism's formalism has anything like the expressive power of possible worlds semantics, different restrictions on its formal frames should validate different modal formulas. In this section, I ask the reader to leave $\mathbb{A}$ and $\mathbb{W}$ uninterpreted and construe $\mathbb{P}$ as an attenuated notion of parthood. With this attenuated notion, we may say that a "property" is "part" of another without assuming that the relation obeys the axioms or theorems of the property mereology in section v. I will show how formal restrictions on $\mathbb{P}$ validate the characteristic axiom schemata of some normal modal systems. For each axiom schema, we place a restriction on $\mathbb{P}$ and abstract from the other potential restrictions. This is an exercise in pure semantics. In the next section, I will interpret $\mathbb{P}$ as the relation of property parthood in an applied semantics for metaphysical modality.

Principle (v) validates the (K) axiom:[41]

(K) $\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$

For $\Box(\varphi \supset \psi)$ is true just in case the property $[\varphi \supset \psi]$ is part of $\mathbb{W}$. Now suppose that $[\varphi]$ is part of $\mathbb{W}$ (that is, that $\Box\varphi$ is true). Then $[\psi]$ is also part of $\mathbb{W}$, by (v). As a result, $\Box\psi$ is true, by (N). So (K) is valid when $\mathbb{P}$ obeys (v).

A property is consistent only if at most one of either $[\varphi]$ or $[\neg\varphi]$ is part of it. $\mathbb{W}$'s consistency secures the following:

(D) $\Box\varphi \supset \Diamond\varphi$

Given $\mathbb{W}$'s consistency, if $[\varphi]$ is part of $\mathbb{W}$ (that is, if $\Box\varphi$ is true, by (N)), then $[\neg\varphi]$ is not part of $\mathbb{W}$ (that is, $\Diamond\varphi$ is true, by (P)). So (D) is valid on frames in which $\mathbb{W}$ is consistent.

I will say that $\mathbb{P}$ is *connected* when $[\varphi]$ is part of $\mathbb{A}$ if $[\varphi]$ is part of $\mathbb{W}$. Connectedness is associated with the following:

(T) $\Box\varphi \supset \varphi$

[41] Strictly speaking, (K) is an axiom schema whose instances are axioms in the K-system. I will often talk as if various axiom schemata are axioms, as if a schema is valid when all of its instances are, and as if a schema is true when its instances are all true.

Suppose that $[\varphi]$ is part of $\mathbb{W}$. Given $\mathbb{P}$'s connectedness, $[\varphi]$ is then part of $\mathbb{A}$. Hence, if $[\varphi]$ is part of $\mathbb{W}$ and $\Box\varphi$ is true (by (N)), then $[\varphi]$ is part of $\mathbb{A}$ and φ is true (by (A)). So (T) is valid on connected frames.

The next three restrictions are new, and I will explain them further in the next section. The first of these requires a definition: a property is *ininclusive* just in case whenever it has a property as a part, the property of being such that it has that property as a part is itself a part of that property. For example, if $\mathbb{W}$ is ininclusive and $[\varphi]$ is part of it, then being such that $[\varphi]$ is part of $\mathbb{W}$ is itself part of $\mathbb{W}$. If $\mathbb{W}$ is ininclusive, the following axiom schema is valid:

(S4) $\quad \Box\varphi \supset \Box\Box\varphi$

Suppose that $\mathbb{W}$ is ininclusive in $\mathbb{P}$. If $[\varphi]$ is part of $\mathbb{W}$, so is the property of being such that $[\varphi]$ is part of $\mathbb{W}$, written $[[\varphi] < \mathbb{W}]$. According to (N), if $[\varphi]$ is part of $\mathbb{W}$, $\Box\varphi$ is true. So if $[[\varphi] < \mathbb{W}]$ is itself part of $\mathbb{W}$, then, in a sense, φ's being necessary is itself part of $\mathbb{W}$. (N) kicks in once more and we conclude that $\Box\Box\varphi$ is true. (S4) is valid on ininclusive frames (that is, frames where $\mathbb{W}$ is ininclusive).

Let us call the following condition on $\mathbb{P}$ *fortification*: if $[\varphi]$ is part of $\mathbb{A}$ then $[\neg\varphi]$'s not being part of $\mathbb{W}$ is itself part of $\mathbb{W}$. When $\mathbb{P}$ is fortified, the following schema is valid:

(B) $\quad \varphi \supset \Box\Diamond\varphi$

Suppose $[\varphi]$ is part of $\mathbb{A}$. By (A), φ is true. Then, by fortification, $[\neg\varphi]$'s not being part of $\mathbb{W}$ is part of $\mathbb{W}$. So, by (P), φ's being possible is part of $\mathbb{W}$. Then, by (N), $\Box\Diamond\varphi$ is true. Hence, (B) is valid on fortified frames.

Finally, call a property *inexclusive* when it meets the following condition: if a property is not part of it, then being such that the property is not part of it is itself part of it. If $\mathbb{W}$ is inexclusive, then if $[\varphi]$ is not part of $\mathbb{W}$, the property of being such that $[\varphi]$ is not part of $\mathbb{W}$ is itself part of $\mathbb{W}$. The axiom schema below is valid when $\mathbb{W}$ is inexclusive in $\mathbb{P}$:

(S5) $\quad \Diamond\varphi \supset \Box\Diamond\varphi$

Let us suppose that $\mathbb{P}$ is inexclusive (that is, that $\mathbb{W}$ is inexclusive in $\mathbb{P}$). If $[\neg\varphi]$ is not part of $\mathbb{W}$, then $\Diamond\varphi$ is true, by (P). This is what we find in (S5)'s antecedent. Given $\mathbb{W}$'s inexclusivity, if $[\neg\varphi]$ is not part of $\mathbb{W}$, then $[[\neg\varphi] \nless \mathbb{W}]$ (read "being such that $[\neg\varphi]$ is not part of $\mathbb{W}$") is part of $\mathbb{W}$. And if $[[\neg\varphi] \nless \mathbb{W}]$ is part of $\mathbb{W}$, we can infer that it is part of $\mathbb{W}$ that φ is possible, by (P). Then we infer, by (N), that $\Box\Diamond\varphi$ is true. This is what we find in (S5)'s consequent. Therefore, (S5) is valid on inexclusive frames.

VIII

I have shown how (v) validates (K), 𝕎's consistency validates (D), and the connectedness, ininclusivity, fortification, and inexlusivity conditions validate the (T), (S4), (B), and (S5) axioms, respectively. When we interpret 𝔸 as the property of being alpha, 𝕎 as the property of being a world, and ℙ as the parthood relation from section v, there is good though not conclusive reason to think that S5 correctly models metaphysical modality. For as long as we assume that the world behaves classically (as I outlined in section VI), there is reason to believe that ℙ obeys all these restrictions. Below, I explain these restrictions in the context of an applied semantics for metaphysical modality.

(K) and (D) are true if Principle (v) and 𝕎's consistency hold. I have simply assumed that (v) is true and that 𝕎 is consistent. I suppose someone could reject one or the other, but these are reasonable assumptions, nonetheless. Principle (v) seems to follow from our intuitive understanding of the material conditional. For if $[\varphi \supset \psi]$ and $[\varphi]$ are parts of 𝕎, surely $[\psi]$ is. Also, our world exemplifies the property of being a world, and I cannot fathom how an exemplified property could have inconsistent propositional properties as parts.

(T) is true if ℙ is connected, that is, if the propositional properties which are part of 𝕎 are also part of 𝔸. We have reason to think that this condition holds. First, being a world in general (𝕎) is part of being *this* world (𝔸). Second, the transitivity of property parthood is a theorem of the property mereology. Therefore, if $[\varphi]$ is part of 𝕎, and 𝕎 is part of 𝔸, then $[\varphi]$ is part of 𝔸. So on the intended interpretation, we now have good reason to accept (K), (D), and (T).

Let me mention a potentially curious result, however. As part of 𝔸, 𝕎 inherits 𝔸's consistency: if there are no properties in 𝔸 that preclude one another, then *a fortiori* there are no properties in 𝕎 that preclude one another. Given (A), 𝔸's consistency guarantees that there are no true contradictions. Given (N), 𝕎's consistency guarantees that there are no true contradictions necessarily. In a way, whatever secures the truth of the law of non-contradiction also seems to secure its necessary truth.

I introduced the next three restrictions in the previous section. I will explain the first of these restrictions by describing a special feature of 𝔸. Take some propositional property $[p_1]$ which is part of 𝔸. Since $[p_1]$ is part of 𝔸, presumably some true proposition p_2 says that $[p_1]$ is part of 𝔸. So from (A), we infer that p_2's corresponding propositional property $[p_2]$ is also part of 𝔸. Therefore, if $[p_1]$ is part of 𝔸, so is the property of being such that $[p_1]$ is part of 𝔸. The privilege of having parts that concern which properties are parts of a property is generally reserved for meta-properties via Inclusivity. But in virtue of being a property which accounts for all truths, even truths about itself, 𝔸 is ininclusive and has properties that concern which properties are parts of *itself*.

The propositional parts of 𝕎 tell us "what it takes" for any world to exist at all. These parts of 𝕎 concern the preconditions for the existence of a world in general, and I doubt whether the preconditions for worldhood could have been different. What could possibly alter the preconditions for the existence of a world in general? If the answer is "nothing," the preconditions for anything's being a world at all include that 𝕎 has the very parts it has. If [φ] is part of 𝕎, [φ] is part of 𝕎 necessarily. That is, if [φ] is part of 𝕎 then [φ]'s being part of 𝕎 is itself part of 𝕎. Like 𝔸, 𝕎 is ininclusive. (S4) is true if 𝕎 is ininclusive.

On the current picture, 𝕎 is part of 𝔸 and both are consistent. Now suppose φ is true, that [φ] is part of 𝔸. Then it seems reasonable to conclude that the preconditions for worldhood could not have included that φ is false. That is, it is a precondition that there is no such precondition that φ be false. So not only is [¬φ] not part of 𝕎 (due to 𝕎's being part of 𝔸 and 𝔸's consistency). Also, the property of being such that [¬φ] is not part of 𝕎 is itself part of 𝕎. So if [φ] is part of 𝔸, it is part of 𝕎 that [¬φ] is not part of 𝕎. This restriction fortifies the truth as necessarily possible and so secures the truth of (B).

I will introduce the final restriction by describing another special feature of 𝔸. There are truths about which properties are not parts of 𝔸. Given (A), the propositional properties corresponding to these truths are also parts of 𝔸. Suppose that [p_1] is not part of 𝔸. Then, it is true that [p_1] is not part of 𝔸. By (A), then, being such that [p_1] is not part of 𝔸 is itself part of being 𝔸. Some of 𝔸's parts concern which properties are not its parts. So 𝔸 is not only ininclusive but also inexclusive. 𝔸 itself is a denizen of alpha, so there are truths about which properties are parts of 𝔸 and which are not. 𝔸's having or not having a part is part of how our world, alpha, is. In virtue of being the property of being alpha, 𝔸 is self-referential as both an ininclusive and inexclusive property.

𝕎 is also inexclusive, though for different reasons. Earlier, I gave a reason to think that the preconditions for the existence of a world in general are preconditions necessarily. Perhaps there is also reason to think that non-preconditions for the existence of a world are non-preconditions necessarily, that is, that [φ] must not be part of 𝕎 if it is not part of 𝕎. If 𝕎 is the kind of abstract object that could not have been different, then if [¬φ] is not part of 𝕎, then 𝕎's not having [¬φ] as a part is itself part of 𝕎. If there are necessary truths about which properties 𝕎 does not have as parts, then 𝕎 has parts that concern which properties it does not have as parts. If $\mathbb{W}$ is inexclusive like $\mathbb{A}$, then (S5) is true.

On the modal intensionalist's interpretation of the formalism, one can make a decent case that the S5 system is appropriate for modeling metaphysical modality. On that interpretation, there is good reason to believe that $\mathbb{P}$ obeys (v), the consistency of $\mathbb{W}$, connectedness, fortification, and the ininclusivity and inexclusivity of $\mathbb{W}$. If $\mathbb{P}$ is restricted in all these ways, (K), (T), (D), (S4), (B), and (S5) are all true.

IX

Okay, there is another semantics for modal logic. Why should we bother with modal intensionalism? First, it can explain why the truths that happen to be true in every possible world are true in every possible world. There is reason to think, then, that modal intensionalism operates on a more fundamental layer of modal reality than modal extensionalism. Second, there are compelling reasons to think that modal intensionalism operates on a layer of modal reality at which we make many of our modal judgments, not only judgments about what is necessary or possible, but even judgments about possible worlds.

Some propositions are true in every possible world. Why are they so special? Why are *they* true in every possible world?[42] The view of properties I am exploring says that nothing could exemplify a property without exemplifying that property's parts. The property of being a world is no exception. So when a proposition's being the case is part of *being a world*, nothing could exemplify the property of being a world without that proposition's being the case. A given proposition is true in each possible world because each possible world, if actual, would have exemplified the corresponding propositional property in virtue of exemplifying the property of being a world. The necessary truths are true in all possible worlds because the necessary truths correspond to the preconditions for being a world that any possible world would satisfy if it were actual. If *being such that* $2 + 2 = 4$ is part of being a world, for instance, then whichever possible world had been actual would have been a world (and thereby) such that $2 + 2 = 4$.[43]

[42] Michael Jubien nicely expresses the feeling that there should be some deeper explanation for why some propositions are true in every possible world. In *Possibility* (New York: Oxford University Press, 2009), p. 75, Jubien writes: "the fundamental problem is that in world theory, what passes for necessity is in effect just a bunch of parallel 'contingencies'. The theory provides no basis for understanding why these contingencies repeat unremittingly across the board (while others do not)."

[43] The full explanation is more complicated than this and only expressible within the context of an intensional approach to quantified modal logic.

Modal extensionalism defines a proposition's necessity as its truth in all possible worlds. Presumably, there are infinitely many possible worlds. But unless we have some heuristic, how could we know what is true in so many possible worlds?[44] We have neither the time nor cognitive horsepower to survey so many worlds. Modal extensionalism separates the layer of modal reality that provides truth conditions for necessarily true propositions from the ways we make judgments about whether propositions are necessarily true. It separates meaning from judgment.[45] Modal intensionalism helps reunite meaning to judgment and even supplies the heuristics to help us make judgments about infinitely many possible worlds.

Modal intensionalism provides at least two kinds of ways for us to know that something is necessarily or essentially true. First, according to modal intensionalism, we can tell whether a proposition is necessary if we can tell whether the proposition's being the case is part of *being a world*. Even though *being a world* is presumably infinitely complex, we do not need to know about all of its parts in order to know that some property is one of its parts, especially if we grant the respectable conception of worldhood according to which a world is the totality of everything there is.[46] On the basis of knowing that consistency is part of what it takes for there to be a totality of everything there is, for example, we may know that propositional properties corresponding to logical truths are parts of *being a world*. Hence, we can know that many logical truths are necessary.

And if we adopt the totality conception of worldhood, we may know that propositional properties corresponding to numerical truths are parts of *being a world*. Being a totality of things involves facts about which things are and are not identical with one another. Facts about identity and diversity bring along with them the number that numbers the distinct things and the numbers that do not.[47] If there is nothing

[44] Modal intensionalism provides the kind of heuristic that other proposed heuristics exploit in some way. For example, we know that a contradiction is not true in any possible world. So we may infer that the negation of the contradiction is true in all possible worlds. But how do we know that a contradiction is not true in any possible world? We seem to have some idea of what it takes to be a world. We know that consistency is part of what it takes for there to be a world.

[45] This concern differs from Benacerraf-style concerns about our knowledge of abstract objects. The present concern is about the form a loosely interpreted semantics takes, not our epistemic connection to abstract objects. Granted, modal extensionalism was never meant to unite meaning and judgment. But that is all the more reason to want a modal semantics that does, not necessarily to replace modal extensionalism, but to have it at our disposal.

[46] See, for example, Stalnaker, "Possible Worlds," *op. cit.*, pp. 69–70.

[47] This argument involves some controversial steps. I defend the connection between totality and number in my paper "Numerical Idealism."

but an *x* and a *y* and *x* is not *y*, there are exactly two things—numbers less than or greater than 2 do not number the things. One number numbers and the rest fail to number only if they all exist. The existence of numbers is part of what it takes for anything to exist at all. Hence, the numbers exist necessarily. From this, we may derive the necessity of arithmetical truths.

Second, the metaphysical backdrop to modal intensionalism provides another route to modal truths. Many of our modal judgments about natural kinds seem to follow this pattern. We identify some kind K, then partly on the basis of observation we build some feature F into our conception of K. We then judge that *being F* is part of *being K* to express that Ks are essentially F. We judge that *being an animal* is part of *being mammalian*, for example, partly on the basis of defeasible evidence from scientific observation. If *being an animal* is part of *being mammalian*, mammals are animals essentially. So if we have good reason to judge that *being an animal* is part of *being mammalian*, which we do, we have good reason to judge that mammals are animals essentially. We do not seem to survey infinitely many possible mammals when we make such a judgment. But the metaphysical backdrop to modal intensionalism offers a plausible, psychologically realistic story about these judgments and thus helps reunite meaning to judgment.

Modal intensionalism also provides an explanation for how we could know what is true in infinitely many possible worlds without surveying them all individually. When we make judgments about what is true in infinitely many possible worlds, we often do it in the following way. Suppose that for some proposition *p*, *being such that p* is part of *being a world*. Given that nothing could exemplify the property of being a world without exemplifying its parts, we may infer from the fact about parthood that *p* would have been true no matter which possible world had been actual just by virtue of the fact that any one of them would have been a world. Whichever one had been a world would have exemplified the property of being a world and *a fortiori* each property that is part of *being a world*, including the property of being such that *p*. If any possible world would have exemplified the property of being such that *p* if that world had been actual, then, as we say, "*p* is true in all possible worlds." Modal intensionalism supplies the heuristic to help us infer in one fell swoop what is true in all the infinitely many possible worlds. This is one way in which modal intensionalism explains the plausibility and success of modal extensionalism. This kind of explanatory support is exactly what we should expect from a semantics that operates on a metaphysically and epistemologically more basic layer of modal reality.

X

Modal intensionalism is an alternative approach to metaphysical modality. Like modal extensionalism, one may fill in the metaphysical details a number of ways. One might identify abundant properties with platonic universals, concepts, states of affairs, or whatever. One may even provide a fictionalist account of properties to exploit the explanatory resources of modal intensionalism and avoid an ontology of properties. The plausibility of any particular intensional approach depends on how one fills in these details.

There are no soundness or completeness proofs here, and modal intensionalism is decades behind modal extensionalism's formal and philosophical successes. But I am optimistic about modal intensionalism's prospects and its potential applications to areas of logic and philosophy that have thus far relied heavily on possible worlds.

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